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# SERIES IN THE ARC SPECTRUM OF NITROGEN*

By C. C. Kiess

In a previous note[1] I have presented series regularities which occur among the infrared lines observed in the spectrum emitted by a nitrogen-filled discharge tube. These lines are also easily observed in the spectra of arcs[2] and sparks[3] in air between metallic electrodes. These series regularities were attributed to the quintet system of the spark spectrum of nitrogen because the term-separations involved seemed to follow Landé's interval rule more closely than on the assumption that they belonged to the quartet system of the arc spectrum, no other criterion being at hand.

Quite recently Fowler[4] has published an analysis of the spark spectrum of nitrogen limited to the wave lengths comprised between 5961A and 3006A. The lines classified by him belong exclusively to the singlet and triplet series systems. Similar results, though not so extensive, have also been found by Croze.[5] It is important to point out that none of the terms associated with the infrared multiplets have been found to occur in the analyses of Fowler and of Croze.

The ordinary methods of exciting the spectra of nitrogen yield band spectra which are attributed to various molecular states, and a line spectrum which is generally regarded as arising from ionized atoms. The spectrum of the neutral atom, if it occurs when condensed and uncondensed discharges pass through a vacuum tube, is apparently lost

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[1] Kiess, Science, *60*, p. 249; 1924.

[2] Kiess, Publ. Amer. Astron. Soc., *4*, pp. 170, 363; 1923.

[3] Merrill, Astrophys. Journ., *51*, p. 236; 1920.

[4] Proc. Roy. Soc., A *107*, p. 31; 1925.

[5] Compt. Rend. *179*, p. 1603; 1924; *180*, p. 277; 1925.

in the band spectra which are always emitted at the same time. In the glow behind the cathode of a canal-ray tube Hardtke[6] observed some lines which he attributed to the arc spectrum of nitrogen, but he did not measure these lines with any great precision.

In a recently published paper, Merton and Pilley[7] describe an experimental procedure for exciting the arc spectrum of nitrogen. Their list of wave lengths, which are quite accurately measured, extends from 7468A in the deep red to 3437A in the ultraviolet. Many of their lines agree with those of Hardtke. The first three lines of their list are identical with lines previously published by Merrill[8] and by the writer.[9] An inspection of the vacuum wave numbers which Merton and Pilley give with their lines revealed the existence of wave-number differences identical with those occurring in the infrared multiplets published in my preliminary note. This fact suggested that the lines of Merton and Pilley and those presented by the writer several years ago to the American Astronomical Society all belong to one spectrum, and led to the analysis of the arc spectrum of nitrogen presented below.

As shown by Fowler, and also by Croze, the series regularities of the spark spectrum of nitrogen are of odd multiplicity. According to the spectroscopic displacement law we should expect series structures of even multiplicity to occur in the arc spectrum. Such is the case. The series regularities presented in the accompanying tables are members of the doublet and quartet systems. In each system the lowest term so far found is a P-term. Above these come P′, S, and D-terms, and in the case of the quartet system, P, D′ (?) and F-terms also.

The low triple P-term of the quartet system is characterized by the separations 46.71 and 33.75. This term combines with an S-term to give the triplet 7468.7A, 7442.6A, 7423.9A. It also combines with a four-fold D-term between whose components the wave-number differences 50.98, 37.35, and 22.59 occur. Two other higher P-terms combine with this common D-term. The three DP multiplets are given in Table 1. The lines of the multiplets which represent the transition $D_4$ $P_3$ (the subscripts being the inner-quantum numbers) are very closely represented by a Ritz formula of the type:

$$\nu_m = nD_4 - mP_3 = nD_4 - \frac{R_N}{\{m+a-b(nD_4-\nu)\}^2}$$

[6] Ann. der Physik, *56*, p. 363; 1918.
[7] Proc. Roy. Soc., A *107*, p. 411; 1925.
[8] Loc cit.
[9] Loc cit.

Here, the Rydberg constant calculated for neutral nitrogen is $R_N = 109732.86$; $a = -0.139841$; $b = +1.86113 \times 10^{-6}$; and $nD_4 = 22461.90$. This formula represents the observed values of $\nu_2$, $\nu_4$, and $\nu_5$, with the residuals $(O\text{-}C)$ : +5.36, +0.11, and −0.19. The multiplet corresponding to the value $m = 3$ lies in the infrared and has not been

TABLE 1. *Quartet PD Multiplets*

| Terms | $nD_1$ | | $nD_3$ | | $nD_2$ | | $nD_4$ |
|---|---|---|---|---|---|---|---|
| | 22572.82 | *22.59* | 22550.23 | *37.35* | 22512.88 | *50.98* | 22461.90 |
| $2P_3$ | | | ——— | | 8718.99 (1) | | 8680.35 (3) |
| 33979.01 | | | *11428.78* | | 11466.08 | | 11517.11 |
| *46.71* | | | | | | | |
| $2P_2$ | 8729.07 (1/2) | | 8711.87 (1) | | 8683.61 (2) | | |
| 34025.72 | 11452.84 | | 11475.44 | | 11512.79 | | |
| *33.75* | | | | | | | |
| $2P_1$ | 8703.42 (1) | | 8686.38 (1) | | | | |
| 34059.47 | 11486.59 | | 11509.12 | | | | |
| $4P_3$ | | | ——— | | 6622.55 (00) | | 6644.97 (7) |
| 7417.09 | | | *15133.14* | | 15095.77 | | 15044.84 |
| *70.12* | | | | | | | |
| $4P_2$ | 6626.98 (0) | | 6636.94 (0) | | 6653.46 (4) | | |
| 7487.21 | 15085.67 | | 15063.04 | | 15025.65 | | |
| *44.34* | | | | | | | |
| $4P_1$ | 6646.52 (2) | | 6656.53 (0) | | | | |
| 7531.55 | 15041.32 | | 15018.72 | | | | |
| $5P_3$ | | | ——— | | ——— | | 5616.54 (4) |
| 4662.36 | | | *17887.87* | | *17850.52* | | 17799.54 |
| *72.05* | | | | | | | |
| $5P_2$ | ——— | | 5611.36 (00) | | 5623.22 (3) | | |
| 4734.41 | *17838.41* | | 17816.06 | | 17778.49 | | |
| *44.66* | | | | | | | |
| $5P_1$ | 5618.18 (00) | | 5625.40 (00) | | | | |
| 4779.07 | 17794.44 | | 17771.40 | | | | |

observed. The term 3P is indicated in Fig. 1 by an open circle, and the permissible combinations between it and other terms are shown by dotted lines. The multiplets corresponding to higher values of $m$ have likewise not been observed, owing, no doubt, to the faintness of the lines. For $m = 1$ the series member would lie in the extreme ultraviolet with wave lengths approximating 760A. Millikan and Bowen[10] have observed some lines in this region but it is not certain that any of these belong to the multiplet in question.

[10] Phys. Rev., *23*, p. 9; 1924.

The P-terms combine with a common P′-term and a common S-term to form the PP′ multiplets and PS triplets given in Tables 2 and 3. Lines not observed because of faintness are in *italics*. A DF multiplet

TABLE 2. *Quartet PP′ Multiplets*

| Terms | $P'_1$ | | $P'_2$ | | $P'_3$ |
|---|---|---|---|---|---|
| | 21868.50 | *18.38* | 21850.12 | *38.36* | 21811.76 |
| $2P_3$ | | | 8242.47 (3) | | 8216.46 (5) |
| 33979.01 | | | 12128.96 | | 12167.36 |
| *46.71* | | | | | |
| $2P_2$ | 8223.28 (3) | | 8210.94 (2) | | 8185.05 (3) |
| 34025.72 | 12157.28 | | 12175.53 | | 12213.85 |
| *33.75* | | | | | |
| $2P_1$ | 8200.59 (1) | | 8188.16 (3) | | |
| 34059.47 | 12190.90 | | 12209.41 | | |
| $4P_3$ | | | 6926.70 (2) | | 6945.18 (3) |
| 7417.09 | | | 14432.92 | | 14394.51 |
| *70.12* | | | | | |
| $4P_2$ | ——— | | ——— | | ——— |
| 7487.21 | *14381.29* | | *14362.91* | | *14324.55* |
| *44.34* | | | | | |
| $4P_1$ | ——— | | ——— | | |
| 7531.55 | *14336.95* | | *14318.57* | | |
| $5P_3$ | | | ——— | | 5829.52 (2) |
| 4662.36 | | | *17187.76* | | 17149.33 |
| *72.05* | | | | | |
| $5P_2$ | ——— | | ——— | | ——— |
| 4734.41 | *17034.09* | | *17115.71* | | *17077.35* |
| *44.66* | | | | | |
| $5P_1$ | ——— | | ——— | | |
| 4779.07 | *17089.43* | | *17071.05* | | |

TABLE 3. *Quartet System PS Triplets*

| | $nP_j$ | | | | |
|---|---|---|---|---|---|
| | 7423.88 (3) | | 7442.56 (4) | | 7468.74 (5) |
| | 13466.34 | *33.80* | 13432.54 | *47.07* | 13385.47 |
| | ——— | | ——— | | ——— |
| $S_3$ | | | | | |
| 20593.28 | *13061.73* | *44.34* | *13106.07* | *70.12* | *13276.19* |
| | ——— | | 6303.74 (00) | | 6275.43 (0) |
| | *15814.21* | *45.02* | 15859.23 | *71.52* | 15930.75 |

is given in Table 4. A group of lines between 6480A and 5420A is suspected of constituting a DD′ multiplet, but the lines are faint and

not enough have been observed to establish the multiplet with certainty. Its arrangement is probably that given in Table 4.

TABLE 4. *Quartet DF and DD' Multiplets*

| Terms | $^nD_1$ | | $^nD_2$ | | $^nD_3$ | | $^nD_4$ |
|---|---|---|---|---|---|---|---|
| | 22572.82 | *22.59* | 22550.23 | *37.35* | 22512.88 | *50.98* | 22461.90 |
| $F_5$ | | | | | | | 6482.74 (9) |
| 7040.58 | | | | | | | 15421.32 |
| *56.06* | | | | | | | |
| $F_4$ | | | | | 6484.88 (8) | | ——— |
| 7096.64 | | | | | 15416.24 | | *15365.26* |
| *34.70* | | | | | | | |
| $F_3$ | | | 6483.77 (4) | | 6499.51 (1) | | ——— |
| 7131.34 | | | 15418.87 | | 15381.54 | | *15330.56* |
| *17.74* | | | | | | | |
| $F_2$ | 6481.73 (3) | | 6491.24 (1) | | ——— | | |
| 7159.08 | 15423.73 | | 15401.13 | | *15363.78* | | |
| $D'_4$ | | | | | 6420.63 (2) | | 6441.70 (5) |
| 6942.35 | | | | | 15570.50 | | 15519.57 |
| *114.84* | | | | | | | |
| $D'_3$ | | | 6452.76 (1) | | 6468.30 (1) | | ——— |
| 7057.19 | | | 15492.98 | | 15455.75 | | *15404.77* |
| *66.42* | | | | | | | |
| $D'_2$ | 6471.02 (0) | | 6480.53 (0) | | ——— | | |
| 7123.61 | 15449.25 | | 15426.58 | | *15389.23* | | |
| *−54.13* | | | | | | | |
| $D'_1$ | 6448.41 (1) | | 6457.89 (0) | | | | |
| 7069.48 | 15503.43 | | 15480.66 | | | | |

Landé's[11] interval rule governing the wave-number separations of the successive members of a complex term is stated only as a first approximation to the truth. For multiplets of even multiplicity, this rule is:

$$\Delta\nu_{1,2} : \Delta\nu_{2,3} : \Delta\nu_{3,4} : \cdots = \frac{3}{2} : \frac{5}{2} : \frac{7}{2} : \cdots$$

A comparison with the observed term separations is as follows:

P and P'-terms, 33.75 : 46.71 = 1.5 : 2.08, calculated 1.5 : 2.5
44.34 : 70.12 = 1.5 : 2.37,
44.66 : 72.05 = 1.5 : 2.42,
18.38 : 38.36 = 1.5 : 3.13.

D-term, 22.59 : 37.35 : 50.98 = 1.5 : 2.48 : 3.39, calculated 1.5 : 2.5 : 3.5

F-term, 17.74 : 34.70 : 56.06 = 2.5 : 4.9 : 7.9, calculated 2.5 : 3.5 : 4.5

For the D-term and the two higher P-terms the agreement is very good.

[11] ZS. für Physik, *15*, p. 194; 1923.

For the F-term the intervals are better represented by the ratios 1 : 2 : 3 than by those of Landé's rule.

Fig. 1 shows graphically the distribution of the terms which produce the quartet system multiplets, the term values being those based on the value of the $D_4$-term calculated by means of the Ritz equation. The prominent multiplets of this system which lie in the deep red and infrared are illustrated in Fig. 2, which is enlarged from a spectrogram

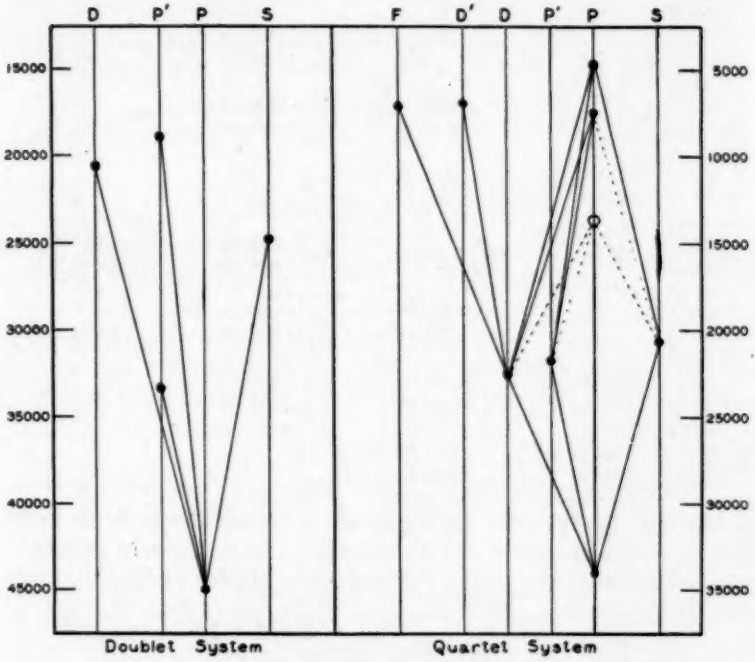


FIG. 1. *Term diagram for the doublet and quartet systems of nitrogen.*

taken with the 7500-lines per inch grating of the Bureau of Standards. The source was a discharge tube containing air at a pressure of about 15 mm.

The doublet system is represented by the line groups given in Table 5. These are combinations of S, P′, and D-terms with a common P-term between whose components occurs the separation $\Delta\nu = 83.12$. The relative positions of these terms are shown in Fig. 1, and in Fig. 2 is shown the infrared PP′ group. The values assigned to the terms are arbitrary as no combinations between the doublet and quartet systems

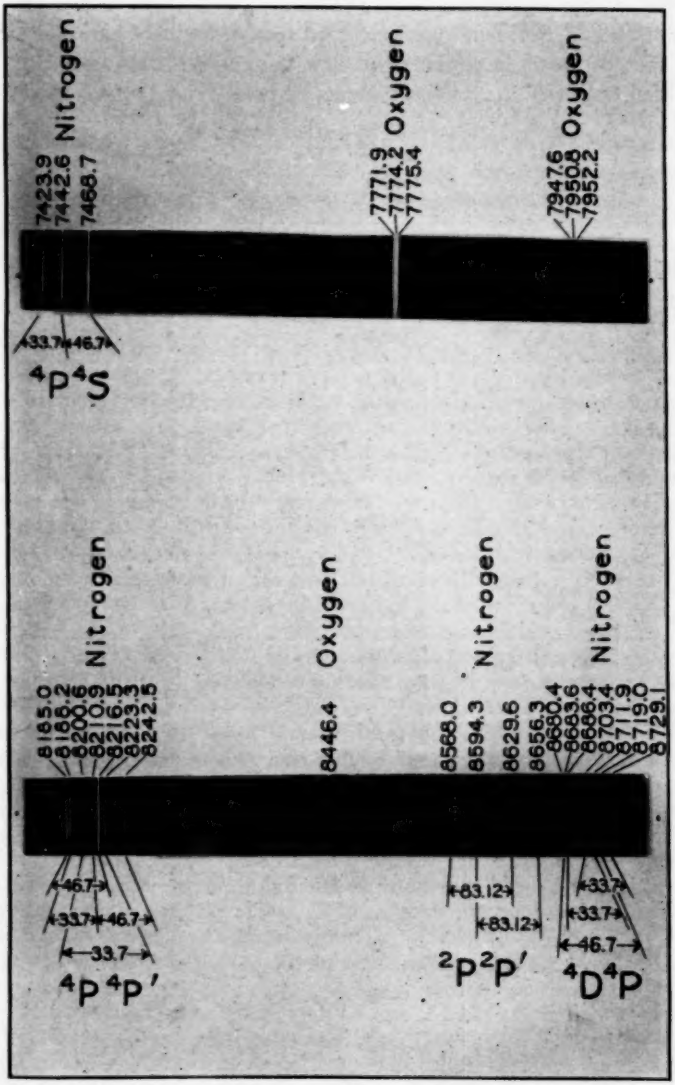


FIG. 2. *Infrared spectrum of air-filled discharge tube.*

have as yet been found. The data are insufficient to establish series as in the quartet system.

Bowen and Millikan[12] have referred to the doublet character of the NI spectrum, and have indicated that the characteristic $\Delta\nu$ is 85.3, as derived from the ultraviolet doublets at 1744A and 1493A. Hopfield's

TABLE 5. *Doublet System Multiplets*

| Terms | $P_1$ | | $P_2$ |
|---|---|---|---|
| | 45083.12 | *83.12* | 45000.00 |
| $a^2P'_2$ | 8568.04 (1) | | 8629.61 (3) |
| 33415.16 | 11668.08 | | 11584.84 |
| | *35.73* | | |
| $a^2P'_1$ | 8594.34 (2) | | 8656.32 (1) |
| 33450.89 | 11632.37 | | 11549.09 |
| $b^2P'_2$ | 3818.19 (1) | | 3830.39 (9) |
| 18900.24 | 26183.02 | | 26099.63 |
| | *26.01* | | |
| $b^2P'_1$ | 3822.00 (5) | | 3834.20 (4) |
| 18926.25 | 26156.92 | | 26073.70 |
| $D_3$ | | | 4109.94 (10) |
| 20675.59 | | | 24324.41 |
| | *23.53* | | |
| $D_2$ | 4099.96 (9) | | 4113.92 (5) |
| 20699.12 | 24383.62 | | 24300.88 |
| $S_1$ | 4914.92 (4) | | 4935.03 (9) |
| 24742.44 | 20340.56 | | 20257.68 |

measurements of nitrogen lines in the extreme ultraviolet extend from 1751A to 835A. Millikan and Bowen also give wave length measurements from 1493A to 660A. Using Hopfield's[13] wave lengths the two doublets in question are

| | | |
|---|---|---|
| 1493.1 (8) | | 1494.9 (7) |
| 66974.8 | *80.7* | 66894.1 |

and,

| | | |
|---|---|---|
| 1740.2 (5) | | 1742.7 (10) |
| 57464.7 | *82.5* | 57382.2 |
| *154.8* | | *157.6* |
| 1744.9 (8) | | 1747.5 (4) |
| 57309.9 | *85.3* | 57224.6 |

[12] Phys. Rev., *24*, p. 209; 1924.
[13] Phys. Rev., *20*, p. 573; 1922.

More recent wave lengths by Hopfield and Leifson[14] are

| | | | | |
|---|---|---|---|---|
| 1492.62 | 66996.29 | | 1742.81 | 57378.60 |
| | | *90.10*, and | | *82.20* |
| 1494.63 | 66906.19 | | 1745.31 | 57296.41 |

The discrepancies that exist between these doublet separations and those given in Table 4 make it very uncertain whether these are related to the P-term with $\Delta\nu = 83.12$. Should they, however, be real, they would then represent combinations between the P-term and a much lower S-term, and a much higher P′-term.

BUREAU OF STANDARDS,
MARCH 30, 1925.

**Photoelectric Properties of thin Films of Alkali Metals.**—This is an immensely elaborate and extensive study of the photoelectric current from thin sublimated films of the alkali metals (all but Li) in their dependence upon six variables: the angle of incidence of the light (from 0° to 80°), its plane of polarization (in or normal to the plane of incidence), its wave length (3800 to 7000 A; the data for different wave lengths are reduced to equal energetic intensities), the applied voltage, the nature of the substratum on which the film is spread, and the amount of metal constituting the film. The alkali metals were introduced into the tube by a series of distillations in vacuo; the glass and the metal parts of the system were freed of gas by intensive bakings and glowings, the vacuum during the work was exceedingly high. From a pool at the bottom of the tube the metal distilled gradually over to the illuminated electrode, a plate mounted centrally within a large collecting cylinder of nickel. If the plate was bare to start with, the photocurrent was initially nil and could be observed gradually rising and changing; its trend could be reversed by heating the plate by a filament mounted behind, and whatever variations were noted in the current the spontaneous growth of the film were accurately retraced during its dissipation by heat. This indicates that the behavior of the film is a unique function of the quantity of matter constituting it,—we may call this its *thickness* for want of exact knowledge of the structure of the film; but no measurements of thickness were made, the film being invisible and doubtless imponderable.

The photoeffect depends to a surprising extent on the substratum of the film. Thus with gold the orientation of the electric vector makes hardly any difference; while with Pt and Ta the effect of the electric vector in the plane of incidence (light incident at 80°) exceeds the effect of the electric vector parallel to the surface by more than twenty-fold. The behavior of films upon Ni, Ag, W, Cu and C is intermediate between these extremes. The experiments on metals other than K were

[14] Astrophys. Jour., *58*, p. 62; 1923.

all carried out with platinum as substratum. These show a remarkable dependence of the two effects (produced with the two orientations of the electric vector) on what has been called the thickness of the film. When the film first becomes perceptible, the two effects are small and not very different (not more different than might be attributed to differences in the percentage of light absorbed). As the film thickens both effects increase, but at different rates, so that eventually the change in the photocurrent when the plane of polarization of the light is rotated is very striking. The effects both pass through maxima and decrease again before the film arrives at its final thickness—i.e. the state in which it is in equilibrium with its vapor—but in this final state the two effects are still very different. It appears therefore that when the amount of photocurrent depends on the orientation of the electric vector of the incident light, this dependence is a characteristic not of the alkali metal itself, as was formerly thought, but of a special thickness of a film of the alkali metal spread upon a special other substance. Another novel result is that a "selective effect" of this kind is not correlated with a maximum in the photocurrent-versus-wave length curve; the wave lengths of maximum photoelectric effectiveness, tabulated by many observers and the object of so much theorizing, are not characteristic of these films nor do they accompany the great differentiation between the effects of the electric vector in its two orientations. The curves of photocurrent vs. wave length mount steadily towards shorter waves; the threshold wave length advances first towards the red, then returns towards the violet, as the film thickens. [H. E. Ives, Bell Telephone Laboratories; Astroph. Jl. *60*, pp. 209-230; 1924]

KARL K. DARROW

**Electron-Transitions in optically excited Mercury Atoms.**—Mercury vapor is irradiated simultaneously by λ2536 and by some other lower frequencies emitted from a water-cooled mercury arc in quartz, and the intensities of various lines which the irradiated vapor emits are compared. The data are interpreted in terms of the Bohr conception, the problem being formulated thus: how does the irradiation of the mercury atoms by light of known constitution affect their distribution among their various possible energy-states? Owing to the feebleness of the rays which the irradiated atoms emit, the irradiating light apparently cannot yet be rendered monochromatic, which is unfortunate; but in this latest work it is somewhat less complex than in the earliest. Perhaps the most clearly interesting result is that when the vapor is mixed with foreign gases, the distribution is changed; e.g. when nitrogen is introduced, the number of atoms in the $2p_3$ state is greatly augmented; the vapor becomes almost opaque to frequencies which only atoms in this state can absorb, and intensely emits frequencies (such as 5461) for which prior attainment of this state is a convenient if not a necessary condition. [R. W. Wood, Johns Hopkins; Proc. Roy. Soc. *106A*, pp. 679-694; 1924]

# THE ORDER OF APPEARANCE OF CERTAIN LINES IN THE SPARK SPECTRA OF CADMIUM AND MAGNESIUM

F. L. Brown and J. W. Beams, Jr

The double refraction of carbondisulphide in an electric field was used so effectively by Abraham and Lemoine[1] and others for the measurement of short time intervals, that Prof. C. M. Sparrow suggested it would be well to try to extend the method to the field of spectroscopy and atomic structure, in an effort to measure the time between the excitation of the atom and the emission of the resulting radiation. The following constitutes a preliminary report on this work.

The apparatus as used by Abraham and Lemoine, Lord Rayleigh and others did not give a complete shutter effect. In the present instance their apparatus was modified by removing the double image prism, and reducing to a minimum the resistance and inductance in the circuit containing the Kerr cell. This modification makes it possible to intercept the light emitted by a spark gap directly in front of the device, although connected in parallel with it. The arrangement is as follows: (See Fig. 1)

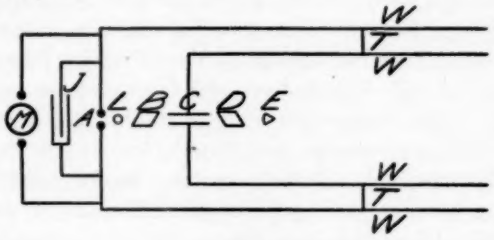

Fig. 1

The Kerr cell $C$ made by immersing two copper plates in a tube of carbondisulphide, is placed between two crossed nicols, $B$ and $D$, their short diagonals making an angle of 45° with the plates. This device will be referred to as the "shutter." An incandescent light $L$ can be viewed by the eye at $E$ through the shutter only when the field is applied. The potential which was supplied by a motor driven static machine $M$ can be regulated by a spark gap $A$ in parallel with the Kerr cell $C$. As the voltage reaches a maximum the light $L$ comes into view, but as soon as the spark discharges across the gap the condenser is

[1] Comptes Rendus, 1899.

also discharged and the light entirely cut off. If now the light *L* be removed and the spark gap itself aligned with the shutter, the light emitted by the discharge of the spark at the gap will reach the shutter after it has closed even though the gap be only a few centimeters in front of the shutter. However, by increasing by a few meters the length of wire in each lead between spark gap and Kerr cell the closing of the shutter can be delayed sufficiently to allow the light to pass through. Apparently the drop of potential occurring first at the gap moves back along the lead wires to the cell.

The long parallel wires *W*, *W*, *W*, *W*, and the cross pieces *T*, *T*, hereafter called the trolley, make it possible to increase or decrease at will the length of this delay-wire within certain limits. If the terminals of the spark are made of the metal whose spectrum is to be examined, it is possible to determine the order of appearance of different bright lines in that spectrum by properly manipulating the trolley. Starting with short lengths of wire, the nicols are crossed so that no light comes through when the gap discharges. As the trolley is moved back lines appear in succession.

The sequence[2] in the appearance of a few of the bright lines of cadmium and magnesium has been observed with a direct vision spectroscope, and photographed with a spectrograph using a 60 degree carbondisulphide prism. This spectrograph was assembled rather hastily for student use but was pressed into service for this work for want of a better one. It will cover with fair definition the range 6600 to 3850 AU on the long dimension of a 4 by 5 plate. However, the shutter used did not transmit wave lengths less than 4200 AU, so that it has not been possible to reach anything outside the visual region. The photographs were taken on Wratten and Wainwright panchromatic plates hypersensitised with ammonia.

For cadmium the lines observed were the red line 6438, the spark doublet 5378 and 5337; and the first members of the sharp series, 5086, 4800 and 4678. In order to make the spark brighter a half-gallon Leyden Jar J is connected to the spark gap using short leads. There were no oscillations in the circuit large enough to affect the shutter. The air lines appeared first with from 3 to 5 meters of wire, (B and S gauge No. 12 bare copper), in each lead. The two Cd spark lines

[2] Schuster and Hemsalech while measuring the duration of lines in metallic sparks noted that the air lines came on first, and that the H and K calcium spark lines seemed to begin after the air lines had gone out. Phil. Trans. *193A*, p. 212, 1900. Schenck in Astrophysical Journal, *14*, 1901, noted that on some of his photographs the Mg arc triplet at 3833 began later than the spark doublet at 4481.

appeared next with about 16 meters of wire in each lead and the arc lines appeared later with about 17.5 meters of wire. The spark was roughly 1 meter in front of the light shutter. Although the delay in seconds can be given only very approximately, there is no question as to the sequence. If to the photographic record are added the visual observations made by both of us, and also observed by other members of the staff here, one would be inclined to place the order of appearance of the metallic lines as follows: first the spark lines[3] $5d_1-4f_1$, $5d_2-4f_2$, followed[4] by 4800, $2p_2-2s$, followed immediately by 5086, $2p_1-2s$ and 4678, $2p_3-2s$ together, last of all line 6438, $2P-3D$. The lack of sensitiveness of the eye to the red as compared to the yellow green may account for the late appearance of the red line, but it certainly appears later than the spark pair.

In the case of magnesium the only lines available are the spark doublet 4481 which Fowler assigns to a combination type; and the first term of the sharp triplet series 5184, $2p_1-2s$, 5172, $2p_2-2s$, and 5167, $2p_3-2s$. Neither the doublet nor the triplet are resolved on the plates, for the lines of the doublet were too close together and an air line at 5180 blurrs the triplet. When, however, the spark is viewed through the direct vision spectroscope it is possible to adjust it so that the short air lines appear at the bottom of the field and the longer metallic lines extend beyond them into the upper part of the field; the triplet is then easily resolved. Again the air lines appeared first with from 3 to 5 meters of wire, the spark lines appeared with 15.5 meters and the triplet with 17 meters. In the photographs the region about 5180 shows a great increase in brightness with anything over 17 meters of wire and this is taken to confirm the appearance of the triplet, at about that time. On account of the extreme brightness of the line 4481, the length of exposures, (60 minutes) and the sensitiveness of the plates in this region, some stray light which does not traverse the Kerr cell directly but is reflected by the condenser plates shows in all the exposures, but the line increases in density in no uncertain manner when the length of wire exceeds 15.5 meters, and to the eye when viewing the lines directly the flashing out of this bright line leaves no doubt as to the time of its appearance.

A preliminary examination of the air lines indicates that the green group at 5002-06 appear before the yellow group at 5667-80 though both are assigned to Nitrogen in Kayser's "Handbuch der Spectroscopie." Other substances are being examined and all show sequence

[3] Salis, Annalen der Physik, *76*, p. 145, 1925.

[4] Revised Paschen Notation.

in the appearance of the lines. The possible effect of the dispersion of the double refraction as the cause of this sequence had been studied. The effect is at most very small and will not explain the observations, for in the case of cadmium the spark lines are on the long wave length side of the sharp series triplet, in magnesium the spark lines are on the short wave-length side of the corresponding triplet yet in both cases the spark lines appear first.

A method for determining the actual time interval between the appearance of different lines is being developed and has been applied in a few cases. Briefly it consists of the following: (See Fig. 2.) *A* is

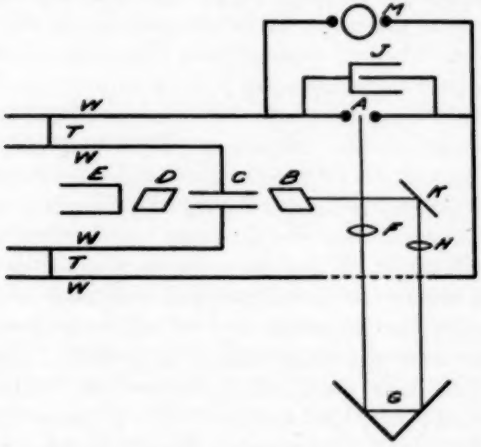


FIG. 2

the spark gap connected to the Kerr Cell *C* and to the static machine *M* as shown in Fig. 1. *F* is a lens which renders the light parallel. *G* is a double mirror for returning the light to the lens *H* which brings it to a focus at the middle point of the shutter. *K* is a plane mirror; *BCD* is the shutter and *E* a direct vision spectroscope. The mirror *G* is mounted on a table on wheels. It can move backwards and forwards and is guided by tracks so as to remain in alignment with the two lenses *F* and *H*. This makes it possible to increase or decrease the distance traversed by the light in going from the spark to the shutter. To measure the delay of the arc lines over the spark lines, the mirror *G* would be brought in as close to *H* as possible and then the trolley moved out till first the spark and finally the arc lines just came through the shutter. *G* is then moved back until both arc and spark lines disappear; it is now brought forward again slowly until the spark lines just appear,

the position being noted, then forward again until the arc lines just appear and this position noted. The difference in light path divided by the velocity of light will give the delay in seconds. The observations have been checked by noting the mirror settings when the lines just disappear. The size of the room limits the intervals which can be measured to light paths of 18 meters or about $6\times10^{-8}$ seconds. This method gives as a provisional value for the delay between the Cd spark and arc lines $3\times10^{-8}$ seconds.[5]

The question has been raised as to whether the value of the delay in seconds is a function of the potential gradient at the spark gap. This question cannot be answered conclusively from the present data—however it is possible to state definitely that the sequence in the first appearance of the lines is not changed by altering the electrical constants of the circuit or by using a transformer instead of the static machine.

It might be suggested that for those atoms from which two electrons have been removed it is apparently necessary for the spark lines to be emitted before the arc lines since the first electrons to return will give the spark lines, while second electrons give the arc lines. This would seem to be borne out by the work of Schenck[6] and Milner[7] who found that the arc lines persisted much longer than the spark lines. However there is no guarantee that all the atoms involved have two or more electrons removed by the first passage of the spark, and since it is the relative first appearances of lines that are being measured and not their duration, the correct explanation of the facts is not so obvious.

Incidentally the method of change in light path has been used to calibrate the trolley in terms of time and indicates that the velocity of the electromagnetic waves in the leads is less than the velocity of light, is not directly proportional to the length of wire used, and depends on the potential, i.e., the length of the spark gap. The work has not been carried far enough to warrant any definite statements regarding these relations or even to allow of ready and certain transformation from lengths of lead wires to intervals of time. But it seems to hold promise of better things when the technique has been more fully developed.

UNIVERSITY OF VIRGINIA,
CHARLOTTESVILLE, VA.,
FEB. 28, 1925.

[5] Since the above was written measurements taken with the same circuit but with an improved mirror system indicate a longer time for this interval.

[6] Loc. cit.

[7] Phil. Trans. *209*A, p. 71, 1909.

**On the Methods of Michelson and of Cotton for the Examination of Optical Systems.**—In Michelson's method a stop having two parallel slits is placed before the objective of the system to be studied. An illuminated single slit serving as a source stands at a suitable distance—in the focus of a collimator objective if plane waves are desired; at the distance of ordinary use in the case of a photographic objective. In the plane conjugate with the source one observes the interference fringes with an ocular or with a microscope. To study all the zones of a system the distance between the components of the double slit is changed, one of the slits remaining on the axis of the system. Faults in the emergent wave-form are revealed by shifts of the fringe system.

But with instruments of large aperture the fringes may become very hard to observe when the moveable slit is over the outer portions of the objective, owing to large path differences. Such difficulties are overcome in Cotton's method in which the components of the double slit are separated by a fixed distance, and the whole is moved across the face of the objective. Thus the system is explored step by step. This simple modification removes the difficulties of observation and permits the use of certain approximations in the calculation of results. Care must always be taken to view the fringes in the plane conjugate with the source.

The mercury arc with suitable filters is a very good source of illumination for the work. Shifts of the fringe system may be measured either with a compensation method or by means of photography. The author states that a path difference of $0.02\lambda$ can be detected. Representative examples of results obtained are given. [M. A. Merland, Revue d'Optique, *3*, pp. 401-413; 1924]

G. W. MOFFITT

**A Rapid Method for Tracing Rays through Optical Systems.**—The classical method of tracing a ray through an optical system requires five or six minutes of time per surface. The author describes a method that is said to be much shorter. By eliminating the parameter $x$ he makes it possible to find sin $i$ when one knows sin $i$ for the preceding surface and sin $u$ between the consecutive surfaces in question. The formula is well adapted for use with a calculating machine. The passage from the angle $u$ for one surface to $u$ for the next surface is made by means of a special nomogram that gives the result by a simple reading. Thus the ray is determined by two independent parameters instead of the eight that figure in the ordinary method. Sixteen operations are replaced by three—two multiplications on the arithmometer and a reading of the nomogram. The time is said to be reduced six to ten fold. The precision is that of five place logarithms. Examples are given in the paper which should be consulted for details. [M. G. Slussareff, Revue d'Optique, *3*, pp. 441-458; 1924]

G. W. MOFFITT

# QUANTUM NUMBER RELATIONS IN SERIES SPECTRA

By R. B. Lindsay

## INTRODUCTION

In a previous paper[1] the writer has computed the orbits of the electrons in the Bohr models of the alkali metals using certain simplifying assumptions concerning screening in applying the usual quantum theory method. The results there obtained for the dimensions of the outermost orbits in the models of these ions show fair agreement with the same quantities calculated from crystal data and other sources, and in particular obey the Grimm inequality law for ionic radii. The values obtained for the effective quantum numbers of the orbits of the series electrons (i.e. the $2_1$ orbit in Li, the $3_1$ orbit in Na, etc.) were not in quantitative agreement with those calculated from the spectral terms, but it was shown that by taking account of the polarization of the inner portion of the atom by the outer electron, the actual effective quantum numbers could be attained to a very close approximation. This has led the writer to attempt the calculation of the orbits in certain other atoms and the computation of the resulting effective quantum numbers. For this work the so-called "stripped" atoms of light elements (i.e. atoms which have been deprived of all but one valence electron) provide excellent material, since the computations are easiest to make in their case and there are now considerable experimental data with which to test the results. The calculations have been carried out for the entire series of stripped atoms in the third group of the periodic system according to Bohr. These calculations are presented and discussed in Part One.

In the second part of the paper, there are discussed certain empirical relations among the quantum defects of the outer orbits in atoms neutral and ionized, and in particular those in the third group discussed in Part One. An attempt is made to interpret these relations in the light of the Bohr theory.

## PART ONE

It will be worth while here to review briefly the method employed in calculating the effective quantum numbers of the orbits in question. The fundamental assumption concerning the screening action of the

[1] Jour. Math. and Phys. Mass. Inst. of Tech., May, 1924. See also Proc. Amer. Phys. Soc. *23*, p. 552 (A8); 1924.

inner groups of orbits is as follows: each electron in the atom will influence the orbits of all the others as a continuous spherical distribution of electricity, the density of which is uniform over every spherical surface about the nucleus and is of such magnitude that the total charge between two such surfaces is equal to the fraction of the charge of the electron corresponding to the fraction of the time which the electron spends between these surfaces during its revolution. In the special case where the orbits are circular and described by the same quantum numbers, this assumption will be interpreted to mean that the mutual screening effect of each electron is taken to be $e/2$ charges, where $e$ is the charge on the electron. On the above assumption an electron in an orbit which lies wholly inside the perihelion of an outer orbit will screen the outer orbit by its whole charge. This method takes no account of the possible orientations of the orbital planes to each other, nor does it consider the possible phase relations which must exist among the electrons in orbits of the same quantum number. For further discussion of these points reference may be made to the first paper mentioned.

For the actual determination of the orbits, the following outline will suffice. Let $V$ denote the potential and $T$ the kinetic energy of the electron. Then if $p_r$ and $p_\varphi$ denote the radial and angular momenta of the electron respectively, we have:

$$T=\frac{1}{2m}(p_r^2+p_\varphi^2/r^2) \tag{1}$$

where $m$ is the mass of the electron and $r$ its distance from the nucleus. If $E$ is the total energy of the electron,

$$E=V+\frac{1}{2m}(p_r^2+p_\varphi^2/r^2) \tag{2}$$

or

$$p_r^2=2mE-2mV-p_\varphi^2/r^2 \tag{3}$$

Assuming a central field, $V$ is a function of $r$ only and $p_\varphi$ will remain constant during the motion, being by the quantum relation:

$$p_\varphi=\frac{kh}{2\pi} \tag{4}$$

where $k$ is the auxiliary quantum number and $h$ is Planck's constant. Substitution into the radial quantum condition, i.e. $\oint p_r dr=(n-k)h$ yields:

$$\oint p_r dr=\oint\sqrt{2mE-2mV-\frac{k^2h^2}{4\pi^2r^2}}\,dr=(n-k)h \tag{5}$$

where $n$ is the principal quantum number and the integration is conducted over a complete circuit from the smaller of the two roots of the expression under the radical to the larger and back again. For convenience we shall call this expression $Q$.

For orbits of a given type the substitution into (5) of the appropriate value of $E$ and the potential energy function $V$ would yield upon integration the proper integral value for $(n-k)$. In the actual problem confronted, however, the appropriate $E$ and course of $V$ are unknown to begin with, and, in fact, $V$ will depend on the electron orbits themselves. We must then proceed by approximations. In calculating the field we may, to a first approximation, use Kepler ellipses instead of the actual orbits, since for the larger part of each period the electrons move in the outer loops of their orbits where the motion is nearly Keplerian. For the same reason it will be seen that the groups of orbits of larger dimensions will have but little influence on the orbits of smaller dimensions. In fixing the orbits the necessary procedure therefore will be to begin with the groups of lower quantum numbers and for each type of orbit to take for a first approximation what seem to be reasonable values for the effective nuclear charge and effective quantum number corresponding to the outer orbital loop. This will then give a value of the energy $E$ and the dimensions of the corresponding Keplerian orbit, from which $V$ can be calculated according to the preceding screening assumption. The $Q$ function may then be plotted as a function of $r$ and the indicated integration carried out. The deviation of the result from the appropriate integral value of $(n-k)$ for the type of orbit in question will then indicate the direction in which the preliminary assumption regarding the orbit was wrong. We proceed with successive approximations until we find a value of $E$ satisfying equation (5). Knowing the field from point to point, we then compute the actual orbit using the Hamilton-Jacobi equations, readjust the values of $V$ to fit the actual orbit and resubstitute into (5). The approximation process is continued until we have found an orbit which is not only correctly quantized, but also consistent in that it yields on analysis the same $Q$ by which it was quantized. It is found that the approximation process here leads rapidly to the appropriate result, two approximations in general fixing the orbit accurately enough.

In computing the outermost orbits (like the $2_1$ in lithium-like atoms and the $3_1$ in sodium-like atoms, etc.) it is necessary to take into account the polarization induced in the inner portion of the atom by the outer electron. If we assume that all the inner electron groups of the atom in

question have their charges $q$ units in number (the unit charge being $e$) distributed uniformly over a sphere of radius $a$ about the nucleus, the expression for the potential $V$ of the series electron for $r>a$ becomes

$$V=-(N-q)\frac{e^2}{r}-\frac{e^2a}{r^2-a^2} \tag{6}$$

We shall use this equation, however, only for values of $r>r_1$, where $r_1$ is that distance for which the resultant surface density of electricity at the point on the shell nearest the electron has become zero. This value is obtained as the root of the equation:

$$\sigma=\frac{e}{4\pi a^2}\left(q-\frac{a(r+a)}{(r-a)^2}\right)=0 \tag{7}$$

where $\sigma$ is the resultant surface density at the point of the shell nearest the electron. It may then be shown that from $r_1$ to $r_{min}$ we have

$$V=-(N-q)\frac{e^2}{r}-\frac{e^2(d^2-1)}{2a}f(r)\ ,\ a<r<r_1 \tag{8}$$

and

$$V=-N\frac{e^2}{r}+q\frac{e^2}{a}-\frac{e^2(d^2-1)}{2a}f(r)\ ,\ r_{min}<r<a$$

where

$$f(r)=-\frac{1}{b\sqrt{dc}}\left[\frac{\sqrt{b+(d+1)^2}}{d+1}-\frac{\sqrt{b+(d-1)^2}}{d-1}\right],$$

and $b=\dfrac{(cd-1)\,(c-d)}{d}$ and $c=r/a$ while $d=r_1/a$.

In applying this method it must be noted that for values of $r\leqq r'_{max}$, where $r'_{max}$ is the aphelial distance of the next to the outermost orbits in the atom we must take a new polarized shell made up of the electrons of groups still farther in toward the nucleus. In computing $V$ under these conditions the next to the outermost orbits will be assumed to screen in accordance with the original screening assumption and $V$ will then take the form

$$V=-(N-q')\frac{e^2}{r}+S(r)-\frac{e^2(d'^2-1)}{2a'}f_1(r) \tag{9}$$

where $S(r)$ denotes the screening potential due to the electrons not included in the shell of radius $a'$.

As before, it is our object to find the appropriate value of $E$ for the outer electron. When this has been obtained by the method above

described, the effective quantum number is deduced at once from the expression:

$$n^{*2} = \frac{N^{*2}e^2}{2Ea_0} \tag{10}$$

where $N^*$ is the effective nuclear charge on the extreme outer part of the orbit and $a_0$ is the radius of the $1_1$ orbit in the hydrogen atom.

The following table, Table 1, presents the course of the potential energy $V$ as a function of $r$ as calculated above for the outer electron

TABLE I. *Potential Energy V as a function of r*

| Atom $r$ | Na | $Mg^+$ | $Al^{2+}$ | $Si^{3+}$ | $P^{4+}$ | $S^{5+}$ | $Cl^{6+}$ |
|---|---|---|---|---|---|---|---|
| 0.05 | | | 218.00 | | 250.0 | | |
| 0.10 | 77.24 | 86.62 | 95.00 | 103.3 | 112.0 | 120.58 | 129.22 |
| 0.20 | 33.18 | 37.71 | 41.13 | 44.37 | 47.70 | 51.36 | 55.01 |
| 0.30 | 18.64 | 21.30 | 23.10 | 24.56 | 26.46 | 28.49 | 30.55 |
| 0.40 | 12.14 | 13.07 | 13.98 | 14.79 | 16.93 | 19.12 | 21.36 |
| 0.50 | | 8.18 | 9.64 | 11.04 | 12.68 | 14.41 | 16.20 |
| 0.60 | 5.48 | 6.17 | 7.43 | 8.57 | 9.95 | 11.36 | 12.94 |
| 0.80 | 3.16 | 3.90 | 4.78 | 6.26 | 7.26 | 8.38 | 9.55 |
| 1.00 | 2.14 | 3.09 | 3.70 | 4.66 | 5.56 | 6.50 | 7.46 |
| 1.20 | | 2.29 | 3.01 | 3.75 | 4.52 | 5.33 | 6.13 |
| 1.50 | 1.08 | 1.68 | 2.30 | 2.91 | 3.55 | 4.20 | 4.85 |
| 1.80 | | 1.34 | 1.86 | 2.39 | 2.93 | 3.47 | |
| 2.00 | 0.71 | 1.18 | 1.66 | 2.13 | 2.62 | | |
| 2.50 | | 0.91 | 1.30 | 1.68 | | | |
| 3.00 | 0.42 | 0.74 | 1.07 | | | | |
| 4.00 | 0.30 | 0.54 | | | | | |
| 5.00 | 0.23 | | | | | | |

in the stripped atoms in the third group, while Table 2 gives the resulting effective quantum numbers for the $3_1$ orbits in these atoms together with the values obtained from the experimentally determined series terms.

TABLE 2. *Effective quantum numbers*

| Atom | Na | $Mg^+$ | $Al^{2+}$ | $Si^{3+}$ | $P^{4+}$ | $S^{5+}$ | $Cl^{6+}$ |
|---|---|---|---|---|---|---|---|
| $n^*$ calc. | 1.63 | 1.91 | 2.08 | 2.20 | 2.29 | 2.37 | 2.45 |
| $n^*$ obs. | 1.63 | 1.90 | 2.08 | 2.20 | 2.29 | 2.36 | |

The values of $r$ in Table 1 are in multiples of $a_0$, while those of $V$ are in multiples of $e^2/a_0$. The experimental values in Table 2 are taken from Bull. Nat. Res. Council,[2] No. 48 with the exception of that of

[2] "Critical Potentials." K. T. Compton and F. L. Mohler.

$S^{5+}$, which is taken from some very recent work of Millikan and Bowen. Table 3 presents the values obtained in multiples of $a_0$ for the radii or aphelial distances of the inner orbits in the atoms considered.

TABLE 3. *Aphelial distances in terms of* $a_0$

| Atom Orbit | Na | $Mg^+$ | $Al^{2+}$ | $Si^{3+}$ | $P^{4+}$ | $S^{5+}$ | $Cl^{6+}$ |
|---|---|---|---|---|---|---|---|
| $1_1$ | .095 | .087 | .080 | .074 | .069 | .065 | .061 |
| $2_2$ | .54 | .52 | .45 | .41 | .37 | .34 | .31 |
| $2_1$ | 1.15 | .98 | .85 | .75 | .68 | .64 | .60 |

For the sake of the discussion in Part II the following table is also appended, giving the values of $\int_i p_r dr$, $\int_o p_r dr$, $\int_e p_r dr$, the first two of which mean respectively the integral of the radial momentum over the part of the actual $3_1$ orbit which lies *inside* the aphelion of the $2_1$ orbit, and the integral over that part which lies *outside* the aphelion of the $2_1$ orbit. The last, $\int p_r dr$, means the integral of the radial momentum taken over the whole Kepler ellipse which coincides with the actual orbit in its extreme outer part and to which is associated the same energy as the actual orbit. It is of interest to note that $\int p_r dr$ is practically constant for the atoms concerned.

TABLE 4. *Values of quantum integrals*

| Atom | Na | $Mg^+$ | $Al^{2+}$ | $Si^{3+}$ | $P^{4+}$ | $S^{5+}$ | $Cl^{6+}$ |
|---|---|---|---|---|---|---|---|
| $\int_i p_r dr$ | 1.23 | 1.24 | 1.25 | 1.23 | 1.24 | 1.26 | 1.26 |
| $\int_o p_r dr$ | .77 | .76 | .75 | .77 | .76 | .74 | .74 |
| $\int_e p_r dr$ | .63 | .91 | 1.08 | 1.20 | 1.29 | 1.37 | 1.45 |

It may be mentioned here that the writer has carried out calculations similar to the above for the singly ionized alkali atoms $Li^+$, $Na^+$, etc. On account of the lack of experimental data in form for testing, the results will not be presented in this paper. It must also be mentioned that Hartree[3] has made interesting calculations on the spectra of the stripped sodium-like atoms, but these are carried out from a different point of view from that used in the present paper.

## PART TWO

The spectroscopic displacement law[4] has brought to light a very important relationship between the arc spectrum of an element and

[3] D. R. Hartree, Proc. Camb. Phil. Soc., *22*, p. 409; 1924.

[4] See A. Sommerfeld, Atombau und Spektrallimien, 3d Ed. p. 456 ff., particularly tables 41 and 42.

the spark spectrum of the next succeeding element. This law states that the two spectra above mentioned have a similar structure, which leads on the Bohr theory to attributing the arc spectrum to the process of formation of the *neutral* atom and the spark spectrum to that of the *singly ionized* atom.[5] In general, if we call the spectrum produced by the binding of the $(N-p)$th electron in the building up of the atom according to the Bohr conception ($N$ represents the atomic number), a spectrum of the $p$th order, then we shall expect a close relationship between the $p$th order spectrum of one element and the $(p-q)$th order spectrum of the element in the same group with atomic number $q$ units smaller. The work on the multiply ionized elements in the second and third groups (discussed in Part I) has verified this view experimentally. This rule will naturally apply only where we have a succession of elements with a common inner structure and not in the case of those elements where, as in the rare earths, the inner quantum groups themselves are modified as we pass from one element to the next.

Numerical relationships between the series terms of such spectra have been discussed by several authors, among them being Sommerfeld[6] and Fues,[7] but no one seems to have considered these relationships from the standpoint of the quantum defect or difference between the principal and effective quantum numbers for the orbits corresponding to the series terms in question. It is true that the quantum defect has been discussed at length by several authors,[8] but not in this connection.

Confining ourselves for the moment to the $s$-terms of the alkalies and alkaline-earths, if we denote the quantum defect by $q$, we have the ratios indicated in the following table.

TABLE 5. *Ratio of quantum defects*

| Atoms | $Na/Mg^+$ | $K/Ca^+$ | $Rb/Sr^+$ | $Cs/Ba^+$ |
|---|---|---|---|---|
| $q/q'$ | 1.24 | 1.21 | 1.16 | 1.14 |

In this table $Mg^+$, $Ca^+$, etc. denote singly-ionized Mg, Ca, etc. respectively. The quantum defects here considered are, of course, those for the $3_1$, $4_1$, $5_1$, $6_1$, orbits of these atoms respectively. If we examine the

[5] See N. Bohr, Annalen der Physik, *71*, p. 242 ff; 1923; particularly the diagrams at the end of the article.

[6] A. Sommerfeld, loc. cit., p. 462.

[7] E. Fues, Ann. der Physik, *63*, p. 1; 1920.

[8] L. A. Turner, Phil. Mag. Sept., 1924 and Nov., 1924; F. S. Brackett and R. T. Birge, Jour. Opt. Soc. *8*, p. 213; 1924.

ratios for the *s*-terms in the stripped atoms of the third group we find the results indicated in Table 6.

TABLE 6. *Ratio of quantum defects*

| Atoms | $Na/Mg^+$ | $Mg^+/Al^{2+}$ | $Al^{2+}/Si^{3+}$ | $Si^{3+}/P^{4+}$ | $P^{4+}/S^{5+}$ | $S^{5+}/Cl^{6+}$ |
|---|---|---|---|---|---|---|
| $q/q'$ | 1.24 | 1.20 | 1.15 | 1.13 | 1.11 | 1.14 |

In this table the first five ratios are obtained from experimental data, while the last is obtained from the calculations in Part I. There is manifest a close correlation between the ratios in the two tables, which may be expressed by saying that in both cases the quantum defect ratio can be writen in the form

$$q/q' = 1 + f_i \tag{11}$$

where $f_i$ is a proper fraction in the neighborhood of 0.2, but having a slight downward trend suggesting an inverse dependence on the atomic number.

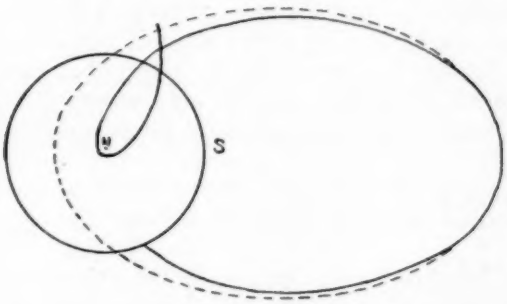


FIG. 1

Before passing to the case of the *p*- and *d*-terms for the preceding elements or seeking for other examples of this approximate ratio constancy, it may be worth while to suggest a brief and roughly approximate interpretation of the matter on the Bohr theory. Referring to Fig. 1, let $N$ denote the nucleus of an atom of the type above-mentioned, while $S$ is the shell which immediately encloses all the inner groups of orbits. In the case of the stripped atoms of the 3d group, for example, $S$ will be measured by the dimensions of the $2_1$ orbits. The eccentric orbit drawn in heavy line represents the real orbit of the outer series electron, while the dotted line represents the Keplerian ellipse of which the outer portion of the real orbit is a part. We may now write the radial quantum condition (5) in the form

$$\frac{1}{h}\int_i p_r dr + \frac{1}{h}\int_o p_r dr = n - k \tag{12}$$

where the two integrals represent the integrals of the radial momentum of the electron in its actual orbit taken over the *inner* part (inside the shell $S$) and *outer* part of its orbit respectively. If $n^*$ is the effective quantum number we have by definition

$$n^* - k = \frac{1}{h}\int_e p_r dr \tag{13}$$

where the integration is now carried out over the dotted line ellipse, which we may call the *equivalent* ellipse. Accordingly we have for the quantum defect

$$q = n - n^* = \frac{1}{h}\left[\int_i p_r dr + \int_o p_r dr - \int_e p_r dr\right] \tag{14}$$

To a rough approximation, since the outer portions of the real orbit and equivalent ellipse coincide, we also have:

$$\int_e p_r dr - \int_o p_r dr = \int_{ei} p_r dr \tag{15}$$

where $\int_{ei} p_r dr$ denotes the integral over the part of the equivalent ellipse which lies inside the shell $S$. Let us now introduce the abbreviations $I_o = \frac{1}{h}\int_o p_r dr$, $I_i = \frac{1}{h}\int_i p_r dr$, etc. We have finally as a rough approximation

$$q = I_i - I_{ei} \tag{16}$$

whence

$$q/q' = \frac{I_i - I_{ei}}{I'_i - I'_{ei}} \tag{17}$$

where the primes indicate the corresponding quantities for the next succeeding atom ionized one degree higher. From our study of the "stripped" atoms in Part I (Table 4) we have to a close approximation for these atoms at least:

$$I_i = I'_i \tag{18}$$

Hence

$$q/q' = 1 + \frac{I'_{ei} - I_{ei}}{I'_i - I'_{ei}} \tag{19}$$

Examination now shows that the quantity $\frac{I'_{ei} - I_{ei}}{I'_i - I'_{ei}}$ in equation (19) is clearly positive, since $I'_{ei} > I_{ei}$ and $I'_i > I'_{ei}$. Moreover, the difference in $I'_{ei}$ and $I_{ei}$ is due mainly to a difference in effective nuclear charge of only *one* unit (the values of $n^*$ for the two differing very little) and hence is small, while the difference between $I'_i$ and $I'_{ei}$, being due largely to the difference between the effective fields in the two cases, which is, of course, very considerable, is rather large. We thus see that the

quantity $\frac{I'_{ei}-I_{ei}}{I'_i-I'_{ei}}$ will be a small proper fraction. This in a rough way leads us from equation (19) to our empirical expression (11). It does not, of course, afford us any indication of how the fraction $f_i$ changes with increasing atomic number. Two possible ways of indicating this present themselves. The first, which is not particularly accurate and rather cumbersome, rests on the fact that to a fair approximation

$$I_{ei} = \frac{1}{2\pi h} \oint_{r_{min}}^{b} \sqrt{-\frac{Z^2}{n^2} + 2\frac{Z}{r} - \frac{k^2}{r^2}}\, dr \tag{20}$$

where $Z$ is the effective nuclear charge on the electron in the extreme outer part of its orbit (and equal to $N-10$ in the case of the stripped atoms considered), and $n$ is the principal quantum number of the orbit in question (i.e. 3 in the case of the stripped atoms). The radius of the shell $S$ is $b$, and the integration is taken from $r_{min}$ to $b$ and back again. The only difference between the actual value of $I_{ei}$ and the integral in (20) lies in the fact that in the former $n^*$ appears where $n$ appears in the above expression. But for small values of $r$ an error in the term $Z^2/n^2$ will not make a very serious error in the evaluation of the integral. The result of the evaluation of (20) is as follows:

$$I_{ei} = \frac{1}{\pi}\left\{\sqrt{2Zb - Z^2b^2/n^2 - k^2} + n \arcsin\left(\frac{Zb/n^2 - 1}{\epsilon}\right) - k \arcsin\left(\frac{Zb - k^2}{Zb\epsilon}\right) + (n-k)\frac{\pi}{2}\right\} \tag{21}$$

where $\epsilon$ is the eccentricity of the ellipse in question. The other quantities are as stated above. It is now possible to calculate the numerator of the fraction in equation (19) for any pair of atoms for which the radii $b$ are known. For the stripped atoms in the third group these radii are, however, given in Table 3 of Part I. The following table gives the values of the $q/q'$ ratios calculated in this way together with the observed ratios.

TABLE 7

| Atoms | $Na/Mg^+$ | $Mg^+/Al^{2+}$ | $Al^{2+}/Si^{3+}$ | $Si^{3+}/P^{4+}$ | $P^{4+}/S^{5+}$ | $S^{5+}/Cl^{6+}$ |
|---|---|---|---|---|---|---|
| $q/q'$ obs. | 1.24 | 1.20 | 1.15 | 1.13 | 1.11 | 1.14 |
| $q/q'$ calc. | 1.26 | 1.19 | 1.17 | 1.16 | 1.19 | 1.24 |

The agreement is not bad until we come to the last two ratios where we should naturally expect the greatest disagreement, since it is for the higher values of $Z$ that the approximation (20) is poorest.

To seek for a more exact evaluation of $q$ and the ratio $q/q'$ we may go back to our expression (14), which we can write in the following form:

$$q=(I_i-I_{ei})+(I_o-I_{eo}) \tag{22}$$

This expresses the quantum defect in two parts, i.e. that contributed by the inner region (*inside* the shell $S$) and that contributed by the outer region. Our previous approximation neglected the latter part entirely. Confining our attention for the moment to the first part, we note that we can express the integral (20) (putting now $n=2$ as a fair approximation, since $n^*$ varies from 1.63 to 2.45) in the form:

$$I_{ei}=\frac{1}{2\pi h}\oint_{r_{min}Z/2}^{hZ/2}\frac{\sqrt{-x^2+4x-1}}{x}dx \tag{23}$$

where $x=Zr/2$ is the new variable. Expanding the integrand in powers of $(x-1)$ and thereby getting a series which converges fairly rapidly, and integrating, we finally arrive at:

$$I_{ei}=\frac{\sqrt{2}Z}{2\pi}(b-r_{min}) \tag{24}$$

in which we neglect terms higher than the first. The expansion being an oscillating one, keeping terms of the second degree would involve retaining those of the third degree as well, which would render the result cumbersome. It happens that the first term yields results which are not bad. Substituting $r_{min}=1/Z(1+\epsilon)$, where $\epsilon$ is the eccentricity of the equivalent ellipse, we have:

$$I=\frac{\sqrt{2}}{2\pi}[(N-10)b-1/(1+\epsilon)] \tag{25}$$

In this expression we can safely use $\epsilon=.86$ as a first approximation for all the elements considered. This leaves only $b$ undetermined.

An examination of the course of $b$ from Table 3 in Part I shows that $b$ can be expressed very accurately in terms of $N$ in the form $A/(N-B)$, where in this case $A=6.70$, and $B=5.17$. If we substitute this expression for $b$ into (25) and utilize 1.24 for the average value of $I_i$ (which is nearly constant), we have for the final approximation to the inner contribution to $q$:

$$I_i-I_{ei}=\frac{8.06-.15N}{N-5.17} \tag{26}$$

The agreement between the observed values (i.e. those computed in Part I) and those calculated by formula (26) is shown in Table 8.

TABLE 8

| $N$ | $I_i-I_\infty$ obs. | $I_i-I_\infty$ calc. |
|---|---|---|
| 11 | 1.10 | 1.10 |
| 12 | .89 | .92 |
| 13 | .76 | .78 |
| 14 | .65 | .67 |
| 15 | .58 | .59 |
| 16 | .52 | .52 |
| 17 | .46 | .46 |

The agreement is good enough to justify us in expecting that $I_i-I_{ei}$ may be expressed in the form $\frac{A+BN}{N-C}$, though the actual values of $A$, $B$, and $C$ may be expected to be in error because of the approximations used. Examination of the observed values indicates that a very good fit is obtained with the form: $\frac{4.49+.022N}{N-6.70}$.

So far as the outer contribution to $q$ is concerned, from the theory just discussed it would seem natural to expect agreement with a formula of the type $\frac{A+BN}{N-C}$ as in the former case. This proves to be true, and if we write

$$I_e-I_\infty=\frac{1.38-.020N}{N-6.70}$$

We get agreement as indicated in the following table.

TABLE 9

| $N$ | $I_e-I_\infty$ obs. | $I_e-I_\infty$ calc. |
|---|---|---|
| 11 | .27 | .27 |
| 12 | .21 | .21 |
| 13 | .16 | .18 |
| 14 | .15 | .15 |
| 15 | .13 | .13 |
| 16 | .11 | .11 |
| 17 | .09 | .10 |

Adding together the two parts of $q$ we have finally:

$$q=\frac{5.87+.002N}{N-6.70} \tag{27}$$

and for the defect ratio:

$$q/q' = 1 + \frac{1}{N - 6.70} \tag{28}$$

The agreement of the observed values and those given by this formula is shown in Table 10.

TABLE 10

| Atoms | $Na/Mg^+$ | $Mg^+/Al^{2+}$ | $Al^{3+}/Si^{3+}$ | $Si^{3+}/P^{4+}$ | $P^{4+}/S^{5+}$ | $S^{5+}/Cl^{6+}$ |
|---|---|---|---|---|---|---|
| $q/q'$ obs. | 1.24 | 1.20 | 1.15 | 1.13 | 1.11 | 1.14 |
| $q/q'$ calc. | 1.23 | 1.19 | 1.16 | 1.14 | 1.12 | 1.11 |

It should be mentioned here that Hartree[9] has also given the form $\frac{A}{N-B}$ for the quantum defect for penetrating orbits in atoms possessing the same inner structure. So far as the present writer is aware, however, he has not yet published a theoretical deduction, nor has he applied the result to the ratios above discussed. (See note below.)

In the preceding discussion we have dealt with the stripped atoms of the third group only and in particular with the *s*-terms. It must be emphasized, however, that the explanation there given will hold for the defect ratios for the *s*-terms in the case of the alkali-ionized alkaline earth metals. This becomes obvious when we consider that in the integral for $I_{ei}$, $Z$ is unrestricted while $n$ can still to a first approximation be taken as 2, since the effective quantum numbers of the outermost orbits in the alkali atoms increase but slowly as we pass from one atom to the next. We therefore expect in this case also an expression for $q/q'$ of the form $1+\frac{1}{N-B}$ where $B$ will, of course, have a different value for each group in the periodic system. Moreover, examination of the approximate form (19) shows that the quantity $\frac{I'_{ei}-I_{ei}}{I'_i-I'_{ei}}$ will not change much for a given series term as we pass from one group to the next in the periodic system. For while $I_{ei}$ increases with increasing principal quantum number, the difference $I'_{ei}-I_{ei}$ will still be dependent mainly on the unit difference in nuclear charge, while the quantity $I'_i-I'_{ei}$ will still be much larger for the reason noted on page (25). Detailed discussion of the various cases will not be given here.

[9] D. R. Hartree, Proc. Camb. Phil. Soc. loc. cit.

Note added during the proof:—Since writing the above, the author has seen for the first time the article by D. R. Hartree in Proc. Roy. Soc. (Nov. 1924) in which a thorough treatment of the quantum defect is given, though on a somewhat different basis from that presented in this paper.

From the mode of derivation of the expression for $q/q'$ we may expect that it will apply only to the case of *penetrating* orbits, and that it will break down for *non-penetrating* orbits. That this is actually the case may be seen from the ratios obtained for the $p$- and $d$-terms in the case of the alkali-ionized alkaline earth metals, exhibited in Table (11).

TABLE 11

| Atoms $q/q'$ | $Na/Mg^+$ | $K/Ca^+$ | $Rb/Sr^+$ | $Cs/Ba^+$ |
|---|---|---|---|---|
| $p$ term | 1.21 | 1.18 | 1.13 | 1.12 |
| $d$ term | .36 | .21 | .14 | .18 |

In this example the $p$-terms still represent penetrating orbits, while the $d$-terms represent non-penetrating orbits. Similar results are shown from an examination of the $p$- and $d$-terms of the stripped atoms above discussed.

All other available examples of penetrating orbits in a pair of atoms possessing a common inner structure but differing in atomic number by unity attest the general validity of the expression (11) for $q/q'$. For example, in the Cu, Ag, Au family, we have $q\text{Cu}/q\text{Zn}^+ = 1.18$, and $q\text{Ag}/q\text{Cd}^+ = 1.13$ for the $s$-terms (the $p$ term ratios show no agreement, for they correspond to non-penetrating orbits.) In the interesting case of *neutral* magnesium and *singly ionized* aluminium, we have for the $s$-term $q\text{Mg}/q\text{Al}^+ = 1.28$, while for the $p$ term the ratio is the same.

Without going into details it is manifest that the quantum defect ratio rule above described ought to find application in the fixing of series terms in previously unanalyzed higher order spectra. In particular, from the known values of $q/q'$ for $Cu:Zn^+$, and $Ag:Cd^+$ it should be possible to compute the $s$ series terms for all the stripped atoms from $Zn^+$ to $Br^{6+}$ and $Cd^+$ to $I^{6+}$ with a fair degree of accuracy. Work of this nature will be presented in a future paper.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
JAN. 26, 1925.

**Thermal Conductivity of Bi in a transverse magnetic Field.**—Measurements of this quantity by a conventional method, upon bismuth rods obtained by melting powdered bismuth and annealing, show that it diminishes by 3.22 per cent when a magnetic field of 5370 gauss, and by about 5 per cent when a field of 6260 gauss is applied; which is in general accord with some and discord with others of the very discrepant earlier measurements. [F. A. Ward, Chelsea Polytechnic; Phil. Mag. *48*, pp. 971-977; 1924] KARL K. DARROW

# INSTRUMENT SECTION

## DESIGN AND CALIBRATION OF A PHONODEIK

By S. Herbert Anderson

### I. Introduction

The term "acoustic oscillograph" may be applied to any instrument designed to receive and record sound waves as they are propagated through the air from any sounding body. The development of such instruments has proceeded along two lines: (1) purely mechanical, (2) mechanico-electrical. In both systems the receiving element is an elastic diaphragm on which the sound waves impinge and produce an oscillating pressure or vibromotive force, which in turn causes the diaphragm to vibrate. In the *first* type the diaphragm is coupled mechanically to a shaft or spindle on which is fastened a small mirror. This system, which will be called a *vibrator*, is given an angular vibration proportional to the linear vibration of the diaphragm. A pencil of light reflected from the mirror vibrates (supposedly) in the same manner as the diaphragm and can be directed, either onto a moving photographic film for a permanent record, or to a rotating mirror and a screen if a visual demonstration is desired. In the *second* type the diaphragm is coupled to an electric circuit and by its vibration changes either the capacitance or inductance in a periodic manner. The alternating current thus generated, after amplification, may be sent, either through the vibrating element of an electric oscillograph and a photographic record made or through a rectifier in series with a galvanometer which gives a measure of the intensity of the sound.

Probably the most satisfactory instrument of the first type for research work is the "Phonodeik" of Prof. D. C. Miller.[1] Dorsey's "Phonelescope"[2] is of the same type, but is designed especially for lecture demonstrations, and embodies certain unique features which insure ease and certainty of manipulation. Lapps' "Undulograph"[3] is a similar instrument, the chief difference being in the vibrator, which consists of a silk thread stretched parallel to the surface of the diaphragm with the mirror fastened of the center. While this greatly

[1] Miller, "The Science of Musical Sounds," 16.
[2] Dorsey's Bulletin No. 1, September, 1921.
[3] Lapp, J.O.S.A. & R.S.I., 7, p. 661; 1923

simplifies construction, it introduces a rather serious defect in that this silk thread is given a transverse vibration by the action of the spring and diaphragm so that the motion of the latter is not fully utilized in rocking the mirror.

Of the second type of sound receivers, special mention should be made of the electrostatic transmitter as modified by Wente[4] as a precision sound receiver; and the electrodynamic transmitter developed by Hewlett.[5] In both of these instruments the diaphragms respond to a wide range of frequencies more uniformly than do the diaphragms of the first type. Wente accomplishes this by air damping of the diaphragm and by tuning the diaphragm to a natural frequency of 10,000 v.p.s., which is just about the upper limit of most sounds to be investigated. In Hewlett's transmitter the diaphragm is nearly aperiodic. However, each instrument must be used with an amplifier in order to secure enough energy to activate an oscillograph or galvanometer. This complicates operation, so that it seems worth while to attempt to improve the first type of instrument.

## II. MODIFICATION OF THE PHONODEIK

In the operation of any oscillograph the natural frequency or frequencies of the vibrating elements must be taken account of. If possible they should be considerably larger than the frequency of any vibration incident on the oscillograph. So far, in the design of *mechanically* coupled sound receivers this condition has not been met. In the investigation of the quality of musical tones, frequencies as high as 10,000 v.p.s. are found. A diaphragm whose fundamental natural frequency is as large as this would be exceedingly stiff, and insensitive to sound waves with their small pressure variation. To secure sufficient sensitivity, fairly flexible diaphragms are used whose fundamental natural frequency is anywhere between 300 and 1000 vibrations per second. This means that the calibration curve will include one or more peaks of resonance.

In Miller's phonodeik the diaphragm is clamped at the end of a resonator horn, which concentrates upon the diaphragm wave fronts of considerably greater area than the diaphragm. The coupling of the diaphragm to the vibrator consists of a fine platinum wire, one end of which is fastened to the center of the diaphragm. It is wrapped over around a small pulley on the spindle and the other end is fastened to

[4] Wente, Phys. Rev. (2) *19*, p. 498; 1922.
[5] Hewlett, Phys. Rev. (2) *19*, p. 52; 1922.

a spring which is under tension. Both the horn and spring have natural modes of vibration, so that with the horn, diaphragm and spring there is a coupled system with *several different frequencies of vibration.* Consequently the calibration curve is very irregular, consisting of numerous peaks and valleys. As Miller points out, any slight modification of the system, as for example might arise from a temperature change, results in an exasperating shift in the peaks and valleys of the curve.

Retaining the principle of Miller's phonodeik, an instrument has been constructed in which the horn and the tension spring have been eliminated, so that only the natural frequencies of the diaphragm need to be considered.

A diagram of the modified phonodeik is shown in Fig. 1. The diaphragm, $D$, is a disk of mica clamped tightly between flat rings of brass with paper gaskets, 5 cms in diameter inside of the clamping rings, 0.011 cm thick and weighing 0.712 gms. To the center of the diaphragm there is cemented a small fork, $F$, made of steel wire 0.025 cm diameter

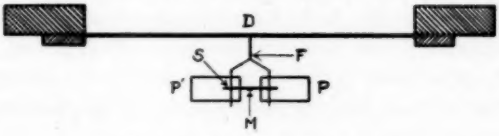


FIG. 1 *Plan of the phonodeik*

with prongs about 0.5 cm long and a distance between them of 0.6 cm. Below the fork are the pole pieces $P.P'$ of a small U-shaped, permanent magnet clamped to the same frame which holds the disk. The pole pieces are ground flat and the magnet so clamped that the surfaces of the pole pieces are parallel to the horizontal diameter of the disk. On the pole pieces is placed a small shaft of steel, $S$, on which is cemented an oscillograph mirror $M$. The magnet is adjusted so that there is a small and uniform pressure between the prongs of the fork and the shaft. When there is a displacement of the diaphragm the vibrator is rolled along by the fork. The pole pieces furnish the surface on which the vibrator rolls and at the same time holds it in position. Its motion is aperiodic, so when the diaphragm vibrates it has an angular vibratory motion strictly proportional to the linear vibratory motion of the center of the disk. Consequently *only the natural frequencies of the disk* need to be considered in the calibration. Moreover, the equivalent mass of the vibrator is small compared with the mass of the diaphragm and fork. The shaft is 0.9 cm long, .0114 cm radius, and weighs 2.8 mg. The mirror is $0.17 \times .06$ cm and weighs 0.3 mg. The equivalent mass

of the vibrator comes out to be 0.0017 gms as compared to 0.712 gms and 0.0137 gms, the masses of the diaphragm and fork respectively.

The magnification of motion with this phonodeik is large. If a screen is placed 50 cm from the mirror, in order that the end of the pencil of light move 1 mm on the screen the diaphragm must be displaced $1.14 \times 10^{-4}$ mm, or the magnification is nearly 9000.

The sensitivity of this instrument will be discussed under the next heading.

## III. CALIBRATION OF THE PHONODEIK

An *absolute* calibration of the phonodeik would require measurement of the angular amplitude of the vibrator when a *known* simple vibromotive force of a certain frequency acts on the diaphragm, this measurement being repeated for each of a sufficient number of different frequencies throughout the range for which the phonodeik is to be used. Fortunately, an absolute calibration is not necessary since the purpose of the phonodeik is to determine the *relative* intensities of the component partial tones of a complex tone. It is sufficient to know the *relative* magnitudes of the different vibromotive forces applied to diaphragm and the angular amplitude of the vibrator corresponding to each vibromotive force.

However, it is not a simple matter to secure vibromotive forces of known relative magnitudes extending over a sufficient range for the calibration. Miller used a set of 61 stopped organ pipes ranging in pitch from $C_2 = 129$ v.p.s. to $C_7 = 4138$ v.p.s. This variety of organ pipe gives a nearly simple tone. These were voiced by the manufacture so as to give tones of equal *loudness*, according to the skilled judgment of the man. Miller states that "this scale is arbitrary and of moderate precision,"[6] but uses it as the best available.

The recent investigation on audition by Fletcher[7], Knudson[8], Kranz[9] and others has shown that the judgment by an individual, or even a group of individuals, that two or more sounds are of equal *loudness* does *not* mean that the sounds are of equal *intensities*. For at least three reasons the ear is an unreliable instrument when used to compare intensities of sounds. In the first place, Fletcher has shown that the sensitivity of the ear to sounds varies greatly with the fre-

[6] Miller, Ibid, p. 145.
[7] Fletcher, Bell Technical Journal, *2*, No. 4, p. 145; 1923.
[8] Knudson, Phys. Rev. *21*, p. 84; 1923.
[9] Kranz, Phys. Rev. *22*, p. 66; 1923.

quency. From the examination of 100 ears, considered to have normal hearing, he finds the average sensitivity, (in terms of pressure variations of the sound waves reaching the ear) to be about 10,000 times greater for a frequency of 2000 v.p.s. than for a frequency of 20 v.p.s. Over the range of frequencies of Miller's set of pipes the average sensitivity varies forty-fold. In the second place, Kranz has shown that in general there is a considerable difference in the sensitivity of the two ears of a person with normal hearing. Furthermore there may be a narrow band of frequencies for which the sensitivity is excessively large or excessively small. In the third place Knudson has shown that the smallest energy change in a sound which will produce a noticeable difference in loudness is about 25 per cent. In other words, the ear is *not* a sensitive instrument for detecting changes of intensities of sound.

In view of this work we must conclude that *relative loudness* of different sounds is quite a different thing from *relative intensities*. In a set of pipes voiced for equal loudness the low pitched pipes will give out much more intense sounds than the high pitched ones; and in particular, there may be a few pipes voiced with a too great intensity or a few with a too small intensity. This latter defect might be remedied by having a large number of persons listen to the pipes and voice them in accordance with their composite judgment. But the former defect would always exist.

Hence, a more reliable method than Miller's must be found for calibrating a phonodeik. Probably the best is the use of a calibrated audiometer[10] as the source of sound for actuating the phonodeik. Such an instrument will emit a simple tone of known intensity and the frequency can be varied over a wide range. With this instrument the calibration could be made in absolute measure.

In the present work an audiometer was not available, so some other method had to be devised. Two different methods were tried which checked each other fairly well.

### FIRST METHOD OF CALIBRATION

Electrically driven tuning forks were used as sources of simple vibromotive forces. For a fixed frequency it was found that the angular amplitudes of the phonodeik vibrator was a linear function of the amplitude of the fork. In the experimental arrangement, the fork was mounted with edges of the prongs toward the diaphragm and 3 cm. from it. The pressure variation at the diaphragm originated in the

[10] Wente, Phys. Rev. *19*, p. 333; 1922.

compressions and rarefactions between the prongs of the fork. For all of the forks (except the lowest pitched) the space between the prongs was nearly the same, 1.5 to 1.8 cm, and the width of the prongs was nearly the same 1.25 to 1.4 cm, so that for a given amplitude of vibration the compression was about the same for all of the forks.[11] A minute droplet of mercury was stuck to the end of one prong of the fork and viewed with a micrometer microscope. When properly illuminated this appeared as a minute bright disk when the fork was still and as a bright line when the fork was vibrating. The double amplitude of vibration was measured with the microscope.

To observe the angular amplitude of the vibrator a narrow pencil of light from a "Pointolite" lamp was directed onto the mirror and after

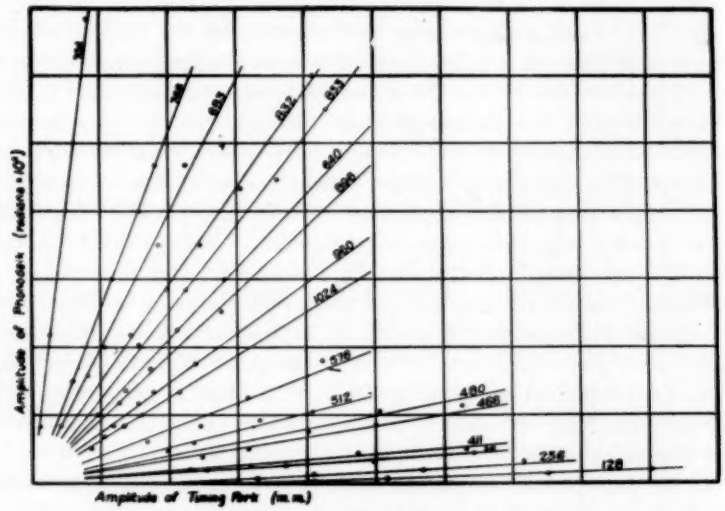


FIG. 2 *Response of phonodeik to tuning forks*

reflection to a millimeter scale set 30 cm from the vibrator. When the phonodeik was vibrating the spot of light on the scale was drawn out into a line and the double amplitude could be easily read off. The amplitude of the fork was controlled by the current through the electromagnet. For a given setting of the rheostat both the amplitude of the fork and of the vibrator remained constant so that the readings could be checked in a very satisfactory way. Amplitudes of the vibrator

[11] See author's note at the end of this paper.

were measured for from four to six different amplitudes of seventeen forks ranging in frequency from $C_2 = 128$ v.p.s. to $C_5 = 1024$ v.p.s.

When angular amplitudes of the phonodeik vibrator are plotted against amplitudes of the fork the points are uniformly distributed about straight lines passing through the origin. Hence the response of the phonodeik is a linear function of the amplitude of the fork and consequently of the vibromotive force at the diaphragm. The slope of the line (angular amplitude of the vibrator per millimeter amplitude of the fork) is a measure of the sensitivity of the phonodeik. The graphs for the seventeen different forks are given in Fig. 2. It will be noticed that for two forks, viz., 128 v.p.s. and 256 v.p.s., the lines do not pass through the origin. This is probably due to errors in observation as the angular amplitudes of the vibrator are very small for these two frequencies. According to Kennelly's[12] theory of vibrating diaphragms the impedance is constant for a given frequency, so that any given vibromotive force should produce a displacement of the diaphragm. His equation for the displacement is

$$x = \frac{F}{(s - m\omega^2) - jr\omega}$$

in which $F$ is a simple harmonic vibromotive force, $x$ the displacement of the center of the diaphragm, $s$ the elastance, $m$ the equivalent mass of the diaphragm, $\omega$ the angular velocity of the simple harmonic motion, and $r$ the resistance coefficient. The linear relation shown in Fig. 2 verifies his theory.

The slopes of the lines of Fig. 2 are plotted as relative sensitivities against the corresponding frequencies and the calibration curve, Fig. 3, so obtained. The small circles are slopes of the lines of Fig. 2, the ordinates being the angular amplitudes of the vibrator in radians per millimeter of the fork and the abscissas are $6 \log \frac{n}{128} \cdot \frac{1}{\log^2}$ where $n$ is the frequency. The curve is observed to have a sharp peak of resonance at a frequency of 704 v.p.s. It is much the same form as the ones obtained by Kennelly for telephone diaphragms. Electrically driven forks were not available for carrying the calibration higher than 1024 v.p.s. by this method. To extend this to higher frequencies another method was used.

[12] Kennelly, Electrical Vibration Instruments, Macmillan, p. 405, 1923.

SECOND METHOD OF CALIBRATION

It is well known that the vibration of a bowed string such as used in a cello or violin agrees very well with the theory of bowed strings as

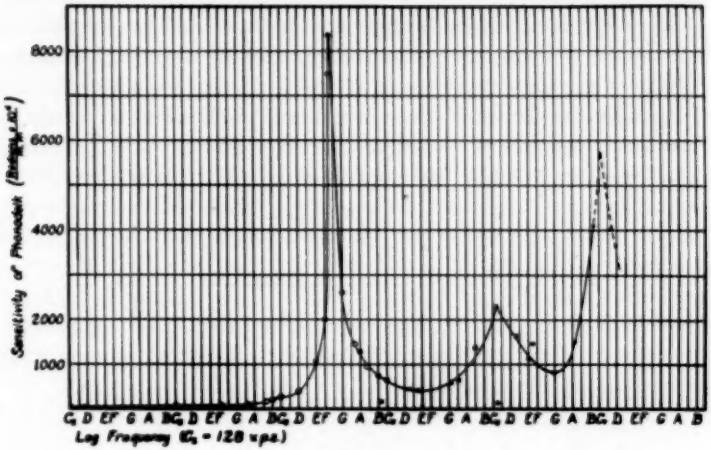


FIG. 3 *Calibration of phonodeik*

given by Helmholtz. Fig. 4 is a shadow picture of a very short segment, about 0.2 mm, at the center of the string, taken by the method originated by Krigar-Menzel and Raps.[13] This is a displacement time graph and the theory requires that the displacement $y$ for any instant $t$ be

$$y = \sum_{m=1}^{m=\infty} \frac{A}{m^2} \sin 2\pi mnt$$

where $m = 1, 3, 5$, etc. This curve was enlarged and analyzed by a Mader's Harmonic Analyzer to the first twenty component partials.

FIG. 4

The results of this analysis are given in table 1. It will be noticed that the analysis shows measurable amplitudes of most of the even partials while the theory requires them to be absent. This is probably due to the influence of the bow which was operated at about one-fourteenth

[13] Krigar-Menzel & Raps, Wied. Ann., *44*, p. 623, 1891.

or one-fifteenth the length of the string, so that the vibration was not symmetrical about the middle point. But this is of no consequence as the even partials were not used. The important thing is that the measured amplitudes of the odd numbered partials agree well with the theory, the average per cent difference being 2.2. We have, then, in a well bowed string a set of at least ten partial, simple vibromotive forces (the odd numbered ones) the frequencies and relative amplitudes of which are known. Moreover these can be impressed on the diaphragm at one and the same time by the complex wave from the bowed string.

TABLE 1. *Relative amplitudes of Partials of a bowed string,* (Fig. 4)

| Partial | Relative Amplitude | | Per Cent Difference |
|---|---|---|---|
| | Theoretical | Measured | |
| 1 | 10000 | 10000 | 0 |
| 2 | 0 | 27 | |
| 3 | 1111 | 1100 | 1 |
| 4 | 0 | 81 | |
| 5 | 400 | 388 | 3 |
| 6 | 0 | 36 | |
| 7 | 204 | 204 | 0 |
| 8 | 0 | 24 | |
| 9 | 123 | 128 | 4 |
| 10 | 0 | 26 | |
| 11 | 83 | 86 | 4 |
| 12 | 0 | 00 | |
| 13 | 59 | 60 | 2 |
| 14 | 0 | 0 | |
| 15 | 44 | 45 | 2 |
| 16 | 0 | 35 | |
| 17 | 34 | 33 | 3 |
| 18 | 0 | 32 | |
| 19 | 28 | 29 | 3 |
| 20 | 0 | 0 | |
| | | Average | 2.2 |

A D-string from a cello was stretched over a heavy steel frame and set before the phonodeik so that the string was horizontal and the center 2 cm from the center of the diaphragm. The sound waves falling upon the diaphragm came from the string alone. Two different pitches were used; (1) one such that the *fifth* partial of the string tone was of the same frequency as the resonance peak of the phonodeik already found by the forks, for which the string was 72 cm long and the frequency of the fundamental 140 v.p.s.; (2) one such that the *third* partial was of the same frequency as the resonance, for which the string

was 48 cm long and the frequency of the fundamental 235 v.p.s. A pencil of light reflected from the mirror of the vibrator was directed to a special type of camera in which a motion picture film was moved uniformly at right angles to the plane of vibration of the pencil of light. Thus a photographic record of the motion of the vibrator was obtained when the diaphragm was actuated by a complex vibromotive

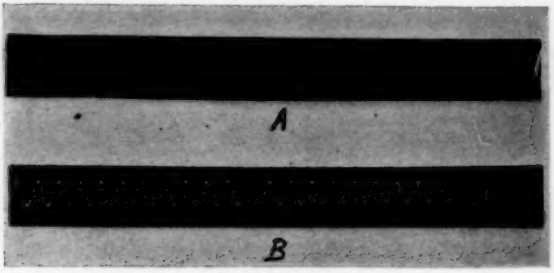


FIG. 5

force, of which the frequencies and relative intensities of the partials are known. The photographs for the two pitches are shown in Fig. 5, A and B respectively. These two curves were enlarged and analyzed for the odd numbered partials, the relative amplitudes being given in

TABLE 2.

| | Rel. amp. of partials of A | | | Rel. amp. of partials of B | | |
|---|---|---|---|---|---|---|
| 1 Partial | 2 From analysis | 3 Reduced to same v.m.f. | 4 Reduced amplitudes ×3.02 | 5 From analysis | 6 Reduced to same v.m.f. | 7 Reduced amplitudes ×9.31 |
| 1 | 12.60 | 12.6 | 38 | 2.35 | 2.35 | 22 |
| 3 | 3.36 | 30.2 | 91 | 100.00 | 900.0 | 8380 |
| 5 | 100.00 | 2500.0 | 7550 | 2.02 | 50.5 | 470 |
| 7 | 1.14 | 55.9 | 169 | 1.40 | 68.6 | 640 |
| 9 | 1.76 | 143.0 | 430 | 3.06 | 248.0 | 2310 |
| 11 | 1.67 | 202.0 | 610 | 1.03 | 124.6 | 1160 |
| 13 | 2.70 | 456.0 | 1380 | 0.52 | 88.0 | 820 |
| 15 | 0.22 | 49.5 | 150 | 0.72 | 162.0 | 1510 |
| 17 | 1.89 | 547.0 | 1650 | 1.53 | 442.5 | 4125 |
| 19 | 1.26 | 455.0 | 1375 | 1.20 | 433.0 | 4030 |

table 2. Columns 2 and 5 respectively give the amplitudes as obtained from the analysis. In columns 3 and 6 the amplitudes are reduced to the same vibromotive force at the diaphragm. That is, from the theory

of the bowed string the amplitude of the third partial is $\frac{1}{3^2}$ times the amplitude of the first. So if the amplitude of the third partial of the curve produced by the phonodeik is multiplied by $3^2$, there is obtained a quantity which *would be* the amplitude of the third *if* the vibromotive force at the diaphragm *was the same* for the third partial as for the first (fundamental). Likewise, the fifth partial is multiplied by $5^2$, the seventh by $7^2$, etc. In column 4 the reduced amplitudes given in column 3 are multiplied by a common factor, 3.02, so that they may be plotted to the same scale as the sensitivities obtained from the forks. For the same purpose the reduced amplitudes of column 6 are multiplied by 9.31. The multiplying factors were selected by trial so that the points would best fit the calibration curve already obtained from the forks. The values of column 4 are shown by squares and those of 7 by crosses in figure 2. The point corresponding to fifth partial of the lower pitched string tone does not come quite at the resonance peak, which simply means the tuning was not as exact as it should have been. (If the frequency had been 141 v.p.s. instead of 140 v.p.s., it would have been exactly at the peak.) The points corresponding to the seventh partial $B_4$, and the fifteenth partial, $C_6$, are considerably off from the curve. No explanation can be offered for the erratic position of $B_4$. The low value of $C_6$ is probably due to the fact that the point of bowing was about one-fifteenth of the length of the string which would cause that partial to have a vanishing amplitude.

Thus by making use of the known modes of vibration of a bowed string it has been possible to extend the range of calibration of the phonodeik over a range of a little more than five octaves, from 128 v.p.s. to 4470 v.p.s., which is the most important range of frequency of musical tones and also of tones of speech. The exact location of the third resonant peak, shown by the dotted portion of curve, is somewhat uncertain. By using a Galton whistle blown at constant pressure it was judged that the peak was just a little above $C_7$. But the sensitivity at that point is not known.

While no attempt has been made to extend the calibration to higher frequencies, the response of the phonodeik has been tested up to a frequency of 8192 v.p.s. A measurable angular amplitude of the vibrator was observed for tones of moderate loudness.

## IV. DISCUSSION OF THE CALIBRATION CURVE

The form of the calibration curve gives some information regarding the character of vibration of the diaphragm. The frequencies of the

three resonance peaks viz., 704, 2092 and 4100, are frequencies of natural modes of vibration of the diaphragm. The ratios of the first and second frequencies to the first are respectively, 2.97 and 5.80. According to the theory of vibration of a circular plate clamped tightly at the edge the gravest or fundamental mode consists in the diaphragm vibrating as a whole with the greatest amplitude at the center and with a node only at the clamping edge. Higher frequencies result from two distinct types of vibration; (1) the nodes are diameters; (2) the nodes are circles with their center at the center of the diaphragm. Resonance peaks of the *phonodeik* (of higher frequency than the first) would not result from the first type of vibration, since the vibrator is coupled to the diaphragm at the center and this would always be one a node. Furthermore this type of vibration seems to be disfavored with the clamping at the edge. Of the second type, when there is one circular node the theoretical ratio of the frequency of this tone to the fundamental is 3.91; when there are two circular nodes the ratio is 8.69. There is considerable difference between the frequency ratio found experimentally and those required by theory. However the values found in this work agree better with those of Miller for a glass diaphragm of the same diameter, viz., 3.28 and 6.72. Kennelly points out that the tightness of the clamping has a marked effect on the character of vibration of a telephone diaphragm. This probably accounts for the discrepancies here observed.

As stated in the introduction, the ideal diaphragm would be one with the frequency of the first resonance higher than that of any vibromotive force incident on the diaphragm and yet with sufficient flexibility. With this point in mind different thicknesses of mica were tried, all 5 cm in diameter. The surprising result was that the resonance peaks had nearly the same frequencies, as shown by table 3. This means

TABLE 3

| Diaphragm thickness | Mass in gms | First resonance | 2nd res. / 1st res. | 3rd res. / 1st res. |
|---|---|---|---|---|
| 0.011 cm | 0.712 | 704 | 2.97 | 5.80 |
| 0.007 cm | 0.462 | 768 | 2.85 | 5.08 |
| 0.005 cm | 0.283 | 820 | 2.52 | 3.78 |

that the elastance of these diaphragms is nearly a linear function of the thickness. If we use Kennelly's theory of vibrating diaphragms, which

is applicable only to the fundamental mode of vibration, the frequency is related to the *equivalent* mass and elastance of the diaphragm by

$$2\pi n_0 = \sqrt{\frac{s}{m}}.$$

He shows that the equivalent mass of an unloaded diaphragm is 0.183 times the true mass. For the *phonodeik* the equivalent mass would be 0.183 × mass of diaphragm + mass of fork + equiv. mass of vibrator. By means of the above equation the elastance of the three diaphragms have been computed and found to be in the ratio

$$11 : 8.9 : 6.45$$

while the thicknesses are in the ratio

$$11 : 7 : 5$$

and the equivalent masses of the vibrating system

$$11 : 7.55 : 4.9.$$

Thus with increasing thickness of mica diaphragms the elastance does not increase quite as rapidly as the equivalent mass, and the frequency decreases slightly, instead of increasing as would be expected. Probably the only method of securing a decidedly higher frequency of resonance would be to use a diaphragm of smaller diameter, which should increase the stiffness and at the same time reduce the mass. However, these changes might reduce the sensitivity so much that the instrument could not be used without a horn.

## V. CONCLUSION

An acoustic oscillograph of the mechanical type has been designed and constructed in which only the natural frequency of the diaphragm needs to be considered, the horn and tension spring being eliminated. The instrument is sensitive enough for the quantitative examination of the quality of musical tones within the range of $C_2 = 128$ v.p.s. to $C_8 = 8192$ v.p.s. Two methods of calibrating the phonodeik have been developed which check each other in a satisfactory manner. The first uses electrically driven tuning forks as sources of simple vibromotive forces and takes as a measure of the sensitivity for a given frequency the linear increase in angular amplitude of the vibrator per unit increase in amplitude of the fork. The second method uses the center section of a bowed string as a source of complex vibromotive force, in which the relative amplitudes of the component partials are known.

Some improvements in design are being tried which will be reported later.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF WASHINGTON,
SEATTLE, WASHINGTON.
JANUARY, 1925,

* Note. Since writing the above my attention has been called to the fact that since the tuning forks and string are not *simple* sources of sound, it is questionable to assume that the vibromotive force at the diaphragm is a linear function of the frequency. Considering the fork as a *double* source the velocity potential for points whose distances from the source are *small* compared to the wave length is given by[14]

$$\phi = \frac{K \cos\theta \cos 2\pi nt}{4\pi r^2}$$

where $K$ is the strength of the source, $r$ the distance from the source, $\theta$ the angle which $r$ makes with the line of vibration of the source and $n$ the frequency of vibration. The vibromotive force will be proportional to

$$\frac{d\phi}{dt} = \frac{-nK \cos\theta \sin 2\pi nt}{2r^2}$$

and hence, is a *linear function of the frequency*, providing $K$ is the same for all forks and the forks are all placed at the same distance from the diaphragm. *This condition was very nearly complied with* except in the case of the lowest fork ($C_2 = 128$ v.p.s.).

In using the string as the source, $K$ will be the same for different segments of the string, corresponding to the different partials respectively, *only* if the length of the segment is large compared with the diameter of the diaphragm. This was approximately true only for partials 1, 3, 5, and 7. For the higher partials corrections should be made. It would seem that above $C_4$ the calibration curve is only of qualitative interest.

**Committee on color terminology.**—Of the 2600 questionnaires sent out by the Committee on Color Terminology of the Optical Society of America, 550 have been answered. Most of these questionnaires have been sent to carefully selected lists of various societies in the fields of research, art, and industry. In the case of the Optical Society, they were sent to the entire membership. To date, only 10 per cent of this membership has sent in replies. Anyone, therefore, who has not replied to the questionnaire is urged to do so at the earliest date. Anyone who has not received a copy should get in touch with Dr. A. Ames, Jr., Wilder Laboratory, Dartmouth College, Hanover, N. H.

The returns received were: 60 per cent from research organizations, 30 per cent from those interested in art and art education, and 10 per cent from those interested directly in the industrial application of color.

A further report will be given to the Optical Society at the Fall Meeting. The results of the questionnaire will then be given, and a report on the color terms used in current literature will be submitted, appended to which there will be a discussion of the pros and cons of the various terms.

[14] Lamb, "Dynamical Theory of Sound," p. 227.

# AN INSTRUMENT FOR THE LABORATORY TESTING OF BINOCULAR TELESCOPES

By G. W. Moffitt and Paul B. Taylor

Devices intended for use in testing the alignment of axes in binocular telescopes have usually been designed to accommodate binoculars of approximately similar dimensions. By thus limiting these devices to instruments of similar form and approximately the same size it is possible to simplify the construction of the testing device and to provide for the rapid routine testing of the type of instrument for which the tester is designed. But in a laboratory where binocular telescopes of widely different sizes and forms are to be studied it becomes a question whether to provide a number of testing devices, each capable of taking a few types, or to provide a single instrument of more complicated design but capable of taking practically any binocular telescope that might come up for test and study. Both plans have certain merits, but in view of the purposes in mind and the local conditions to be considered it seemed more advisable to construct the single instrument of more complicated design.

While testing devices could be arranged to measure directly the angle between the optical axes of a binocular, it is simpler and more convenient to make a direct determination of *the angle between the emergent beams when the incident beams bear some fixed relation, usually parallelism, to each other*. All the devices mentioned below, as well as the new one described here, are built on this principle. One is seldom interested in the angle between the optical axes of a binocular. The thing that is of importance, at least to the user, is the directly measured angle between the emergent beams when the incident beams come from a point at a distance equal to the average object distance in actual use. (Since most binoculars are used in the viewing of distant objects it is usually sufficiently precise to assume parallelism of the incident beams). It is these emergence errors that determine how the axes of a pair of eyes must be oriented with respect to each other in order to use the instrument with binocular vision. Tolerances should be stated in terms of these errors rather than in terms of optical axis alignment errors. If this be done the tolerances will be practically the same for all instruments regardless of magnifying power. If the errors of alignment of the optical axes are ever needed, they can be found quickly by dividing the emergence error by a factor equal to the magnifying power minus unity.

The elements of a tester built by Zeiss[1] are shown in Fig. 1. There are two parallel collimators with illuminated reticules, from which the light passes into the objectives of the instrument under test. On emergence from the eyepieces the light is received by a pair of properly located parallel telescopes. When the binocular is adjusted to bring

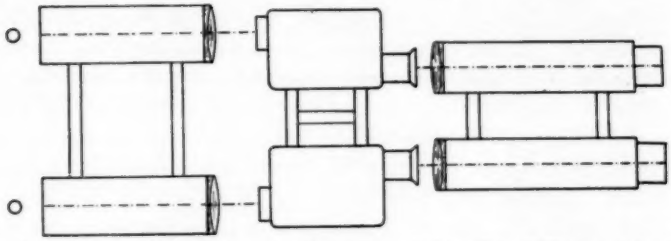

FIG. 1. *One form of binocular tester by Zeiss, composed of parallel collimators and parallel telescopes*

the image of one reticule to the cross wires of the corresponding observing telescope, the image of the other reticule should lie on the cross wires of the other telescope. In order to accommodate binoculars of different objective separations, rhombs and plane parallel plates have been used to offset the beams of light, as indicated in Fig. 2.

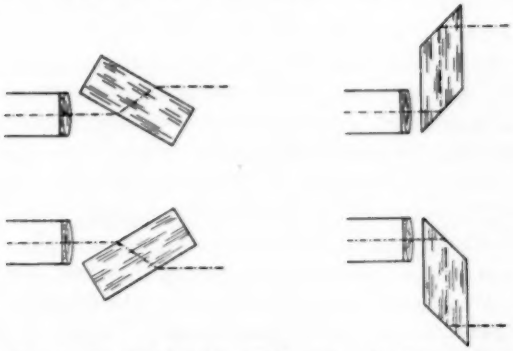

FIG. 2. *Introduction of offset in parallel beams by refraction and by reflections.*

In a British instrument[2] there is but one observing telescope, which may be moved transversely from one tube to the other. The hinge of the binocular is held in V's, and when rotation of the binocular in these

[1] König, Die Fernrohre und Entfernungsmesser, p. 106, Raibaud, Revue d'Optique, *1*, p. 481; 1922.

[2] Trans. Opt. Soc. London, *17*, p. 93, 1916; *20*, pp. 105-108; 1919.

V's does not move the image, that half of the binocular through which the image is seen is in adjustment.

Another instrument that has been used in this country for adjustment during manufacture has a pair of rigid collimators and a pair of rigid telescopes. A holder is provided for the binocular. One body tube of the binocular is clamped by the holder while the other is free to rotate about the hinge. Holder and optical axis are simultaneously adjusted until proper alignment is obtained when the tube is rotated from one collimator to the other.

The plan of a binocular tester used by the Bureau of Standards[3] is patterned after one used by the French Army.[4] In this device the two observing telescopes are replaced by a single large long focus projecting

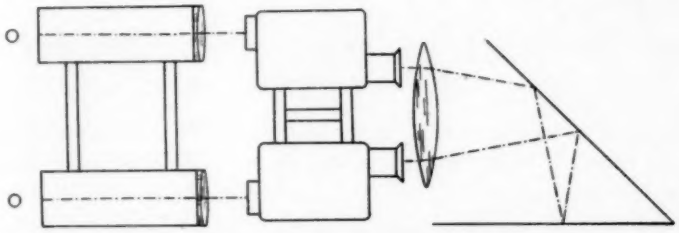

FIG. 3. *French binocular tester, showing two parallel collimators and a single large projection lens with mirror and ground glass screen.*

lens of sufficient diameter to include the beams from both eyepieces of the binocular. By the use of one or more mirrors the position of a ground glass screen can be made suitable for convenient observation while operating the instrument. The two images from a binocular in adjustment will appear superposed. The spherical aberration of the single projecting lens does not interfere, but, since the lens is not achromatic and a filter is used, no judgment of the chromatic corrections of the binocular can be made. This type of instrument is illustrated in Fig. 3.

The two collimators may also be replaced by a single collimator of large aperture,[5] as in Fig. 4. This scheme obviates the necessity of adjustment for parallelism, because proper parallelism is assured when proper focus of the collimator has been obtained. On the other hand, it makes the apparatus bulky and does not permit a difference in the patterns seen through the two tubes of the binocular.

[3] Smith, J.O.S.A., *2-3*, p. 76, 1919.
[4] Raibaud, l.c.
[5] Raibaud, l.c.

The new instrument described here was designed primarily to test practically any binocular telescope in those details peculiar to a binocular instrument, namely, adjustment of the optical axes, equality of magnification, leaning and relative leaning of the images. In addition, however, it will test for quality of imagery, width of field of view, and errors of the reticule—if one be present. In fact, the only properties it will not measure are light transmission and pupil diameters, for the measurement of which separate instruments are normally used.

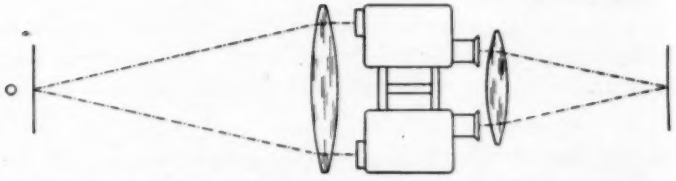

FIG. 4. *Binocular tester with single large collimator lens and single observing lens.*

In order to take the great variety of existing binocular telescopes differing widely in their dimensions and perhaps having various attachments incorporated with them, it was necessary to provide a wide range of adjustment to the parts of the testing instrument. This fact accounts for most of the difficulties encountered in the designing, and leads to

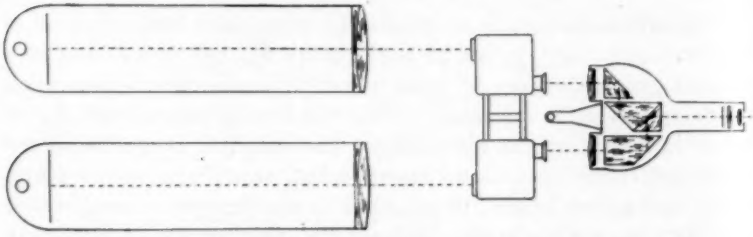

FIG. 5. *Diagram showing the optical system of the new tester.*

most of the departures from the details of instruments already described. In addition to the relatively leisurely work of studying the occasional sample instrument, the tester was to be capable of handling standard binoculars quite rapidly.

These considerations led to the type of instrument shown schematically in Fig. 5, and of which Figs. 6, 7, 8 and 9 are views. Light from the illuminated reticules of the collimators passes through the binocular under examination to the corresponding objectives of the

special observing telescope. The two full-aperture beams are here united by means of suitable prisms and a semi-platinized mirror surface. This superposes the images and a single eyepiece suffices. If the binocular is in accurate adjustment the two images will be concentric, as indicated by the alignment of the radial line segments in the images. If the glass is not so accurately adjusted the images will be displaced relative to each other, and the kind and the amount of this maladjustment may be read on the reticule divisions of the observing telescope. Since the reticule patterns in the two collimators are supplementary to each other, coincidence is observed by the meeting of line segments rather than by the superposition of unbroken lines. Moreover, one knows instantly which image is formed by each tube of the binocular.

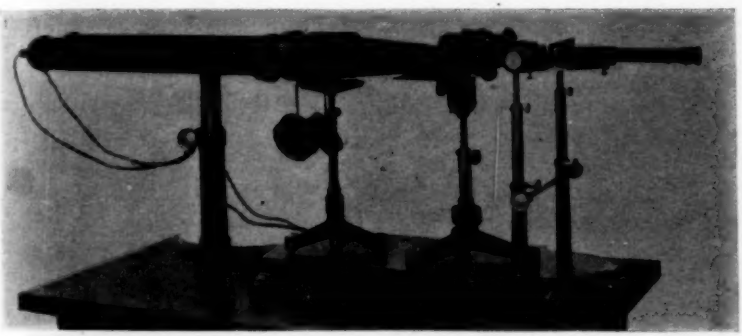

FIG. 6. *Binocular tester with large double telescope in position for test.*

This type of construction leads to a number of advantages. In the first place, there is only one eyepiece. Moreover, careful alignment (that is, to the point of establishing a bisection) is not required, since the maladjustment can be seen at a glance, and read on any part of the reticule scale so long as the image patterns are anywhere on the graduated portion of the reticule. Leaning differences are rendered doubly obvious to the eye because of the proximity of the image lines that ought to be parallel. Differences in image forming quality are also strikingly apparent.

By providing rotational movements for the instrument under examination, and also for the observing telescope, central portions of the collimator and observing telescope fields may always be used. Therefore any faulty definition that may be observed can reasonably be attributed to the binocular under examination. These rotational

movements also provide means for the study of the field performance of binocular telescopes.

The base of the instrument is a heavy casting that is fundamentally two interlocked isosceles triangles. The supporting feet are placed at the corners of one of these triangles. The supporting pillars for the superstructure are firmly attached to the remaining corners. It might be asked why these pillars were not placed directly over the feet. It was necessary to choose between two possibilities. The one was to support the pillars directly over the feet and run the risk of having the heavy instrument platform (the ends of whose track would be unsupported) deflect the casting when rotated in azimuth. The other possibility was to support the instrument with a foot under the pivot of the instrument platform and one at each end of the track on which the outer end of the instrument platform rides. The latter plan was adopted

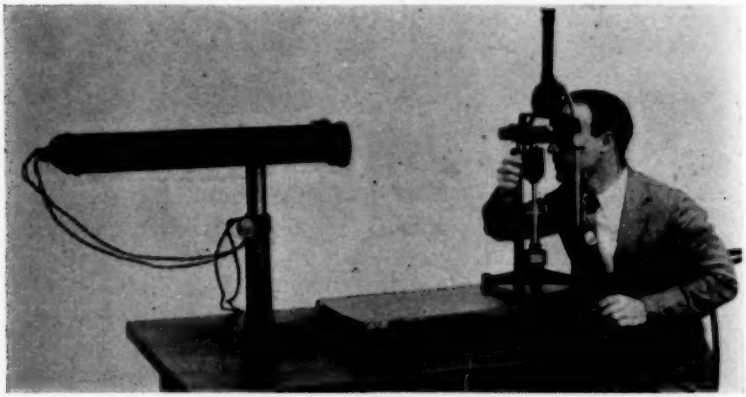

FIG. 7. *Binocular tester, showing observing telescope raised to permit direct observation through the instrument under examination.*

because it would not lead to deflections of the platform or of the optical tubes supported by the pillars, although leaving the optical tubes somewhat more susceptible to vibration. Tests of the base have shown that either plan would have been satisfactory and that the vibration of the optical tubes is seldom noticeable in use. It would obviously not be a safe design for instruments of a more precise character, however. The vertical dimensions of the base casting would probably not satisfy the mechanical engineer, but, while the casting looks rather shallow, it is entirely satisfactory for the purpose. It was desired to keep the

working surface of the instrument platform as low as possible in order not to unduly lengthen the pillars and also to keep the line of sight at a suitable height with respect to the level of the table top on which the tester was to stand.

In detailing the collimator, care was taken to provide a construction that would hold the optical axes rigidly as adjusted. This unit must be designed to resist torsional deflections if the axes are to remain parallel. By pinning the two body tubes firmly to bosses on the figure-of-eight end castings they became torsion resisting members. The lens cells at one end and the reticule cells at the other are mounted directly into the figure-of-eight castings. The tubes are strengthened with interior rings that serve the double purpose of forming good support attachments and also of strengthening the tubes against accidental denting. The whole collimator is carried on a large steel tube that slides in the pillar on the base casting. This provides a considerable range of height adjustment. A spline in the pillar fits a keyway in the supporting tube so that on raising or lowering the collimator, the azimuth may be quite closely maintained. The clamp used is of the contracting band type, a clamp that is much more effective than a set screw would be, as it has a large gripping surface, aligns the sliding tube accurately, and draws it snugly up against the spline.

The collimator objectives are well corrected achromats matched in focal length to within a reasonably small difference. The ones used have an equivalent focal length of 24 inches and a clear aperture of 3 inches. Their centers are 4.75 inches apart. This is a good average value for the separation of the objectives of prismatic binoculars, and the excess diameter of the collimator objectives will take care of all separations and objective sizes that are ordinarily found in hand binoculars.

The collimator reticules are opaque, giving bright lines on a dark field. This seems to be the preferable arrangement for showing up the defects in the quality of the images.

The illumination provided is that of a small automobile lamp placed in a suitable white holder. This gives sufficient brightness for the central part of the image pattern but does not do so for the edges. An illuminating device consisting essentially of a large Gauss eyepiece is being constructed. This will give ample brightness over the entire collimator reticule, and will also serve as an eyepiece for use in focusing the collimator by auto-collimation.

The instrument platform is a rigid bronze casting mounted on a pivot near the observer's end of the instrument, and supported by rollers

running on a track in the other end of the base casting. A divided arc with a vernier is provided for reading the position of the platform in azimuth.

Holders are provided for supporting the usual types of binocular telescope on the instrument platform, and even for those unusual instruments for which provision has not been made it is comparatively easy to arrange a structure of laboratory supports and blocks on the broad expanse of the instrument platform in such a manner as to take care of the exceptional case quite satisfactorily. The most generally

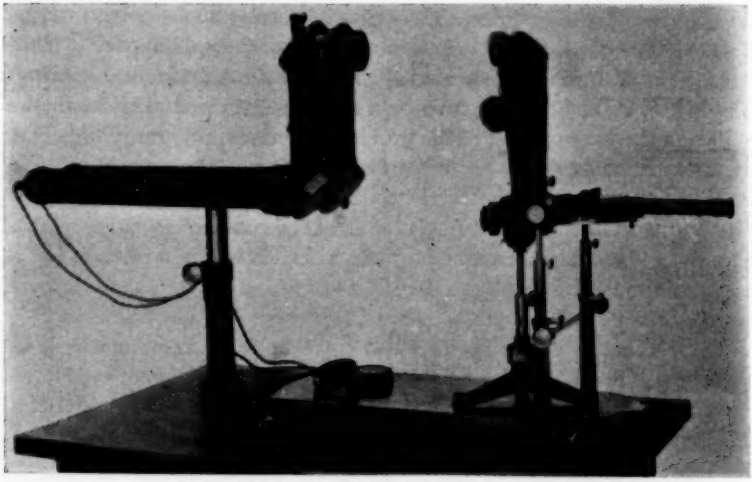

FIG. 8. *Binocular tester with extension arms in vertical position, testing battery commander's telescope.*

useful support is the tripod, on the post of which several devices may be placed. A telescoping tube and rod provide for coarse vertical adjustment while fine adjustment is accomplished by means of the jackscrew at the base of the post. Rotation about a transverse horizontal axis is secured by means of a worm gear. This axis is placed as nearly as possible to the exit pupils of the binocular in order that tilting the instrument will change the position of the exit pupils but slightly with reference to the observing telescope. The tripod may be placed anywhere on the instrument platform, but it is intended that it be so placed that the exit pupil plane of the instrument under test pass through the pivot of the instrument platform. This eliminates objectionable lateral motion of the exit pupils when the platform is rotated

in azimuth. Practically all ordinary instruments can be taken by the fittings that are provided for the top of the tripod post.

The observing telescope is carried by a cradle supported on two adjustable posts sliding in the pillars rising from the base at the sides of the pivoted end of the instrument platform. The sliding rods and the rack and pinion motion provide for a total vertical adjustment of fifteen inches. To constrain the rods to move in unison the pinions were cut in a single rod that extends all the way across from one pillar to the other; this rod being so placed as not to interfere with the apparatus on the instrument platform. Placing the supporting pillars at the sides also helps in this matter of keeping down the possibility of interference of parts.

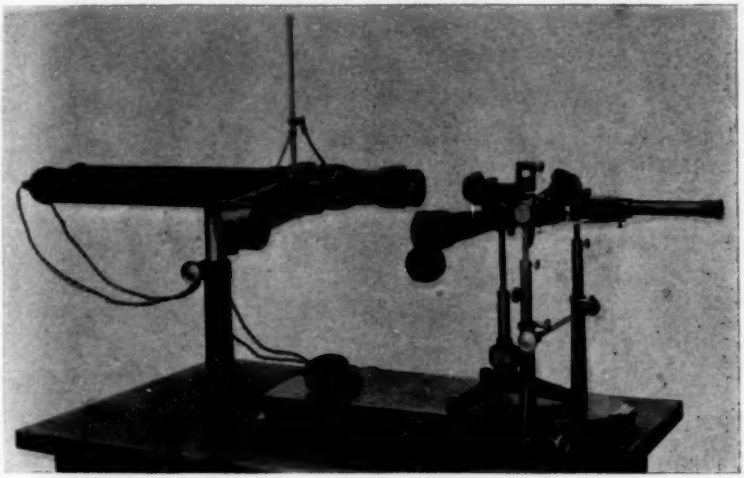

FIG. 9. *Binocular tester with extension arms in the horizontal position, testing battery commander's telescope.*

In order to provide for looking directly into the binocular under test, as is sometimes desirable when aligning it with the collimators, the observing telescope mounting has a transverse horizontal axis about which it may be swung up out of the way, as shown in Fig. 7.

The rotation of the observing telescope in azimuth is intended to be coaxial with that of the instrument platform. A graduated arc with a vernier is provided for reading its position.

The objectives of the observing telescope are about an inch in diameter and are spaced so as to take all separations of binocu'ar eyepieces

corresponding to any natural pupillary distance, and any practical visual field angle. The beam uniting prism system transmits and reflects to about the same degree so that the two images are of approximately equal brightness. The adjustment of the prisms is such that the two images of an infinitely distant pattern coincide precisely and, owing to the matching of the objectives, this coincidence extends over the entire field.

Thus if one looks through the observing telescope without any binocular on the instrument platform the two images of the collimator reticules are seen superposed so that the segments meet to form complete circles and the radial lines to form complete diameters.

FIG. 10. *Appearance of the field of the observing telescope with a binocular under test. The upper image is from the right hand tube. Note the difference in sharpness of the images and the lack of coincidence due to non-parallel emergent beams.*

When a binocular intervenes between collimator and observing telescope the two images will usually be displaced with respect to each other as shown in Fig. 10. These displacements may be read on the reticule of the observing telescope. On this reticule the principal lines are one degree apart (visual field of the binocular under examination). One of the squares is divided up into small 10 minute squares for convenience in reading the emergence errors of the binocular in case actual numerical data is desired. Just below this divided square is a rectangle

1° long by 30′ high, which serves as a tolerance rectangle; that is, if the centers of the two images can be simultaneously brought within this rectangle, and if the emergence error is one of divergence, the glass is satisfactory as far as alignment of axes is concerned. Convergence of emergent beams (which appears in the observing telescope as a horizontal overlapping of the images) should not be permitted at all as it demands a divergence of the eyes of the user—an altogether unnatural condition for the eyes to meet.

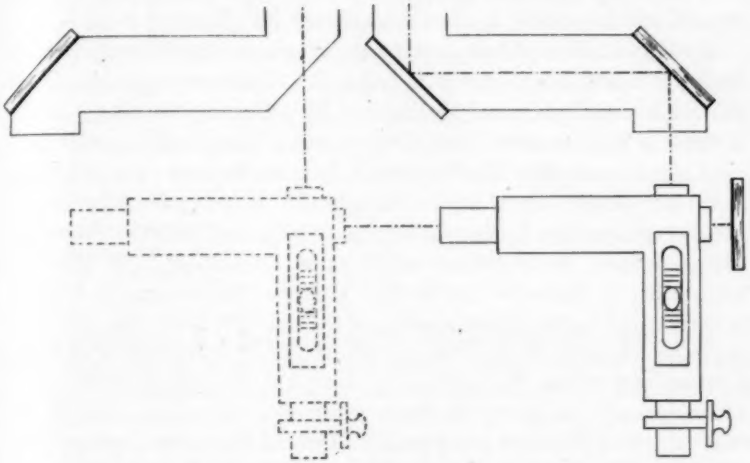

FIG. 11. *Diagram of the device for securing parallelism of beams. The dotted portion indicates the second position of the adjuster.*

For instruments that may have widely separated entrance pupils, such as the battery commander's telescope, a wider separation of parallel beams than can be had with the double collimator alone is required. When the arms of these instruments are in the vertical position the separation of the entrance pupils is about the same as that in a hand binocular, but there is a vertical offset of from 8 to 15 inches. When the arms are extended the vertical offset is small but the separation of the entrance pupils may be as much as 30 inches. To provide for the study of binoculars of this type a set of extension arms has been designed and constructed to fit over the collimator objectives. This attachment is shown in place in Figs. 8 and 9, schematically in Fig. 11. The arms are aluminum castings each carrying a pair of front silvered plane mirrors adjusted to parallelism. Thus light from the collimators

will emerge from the outer ends of the arms parallel to the direction it would take were the arms removed. By rotating the arms and by adjusting the height of collimator, binocular, and observing telescope a position is found for each type of instrument. Practically any modern battery commander's telescope can be examined.

For aligning the collimators and for adjusting the extension arms an auxiliary device, shown in Fig. 11, is very useful. This device consists of two telescopes mounted at approximately right angles, preferably in a single casting, and having a sensitive level bubble with its axis approximately parallel to the axis of one of the telescopes.

The device is first placed so that the cross wires of the telescope having the level bubble are centered on one of the collimator reticules. Then the other telescope is auto-collimated by adjusting the plane mirror in front of its objective. Both of these adjustments will probably proceed simultaneously. The bubble is then either read or adjusted to zero. The device is next moved back until it is facing the other beam, care being taken not to disturb the plane mirror used in auto-collimating the telescope. Auto-collimation is again established, this time by adjustment of the *feet* of the device. It is also simultaneously levelled to correspond to the former reading of the bubble. When this has been done the telescope that is looking into the beam will be parallel to its first position within the error of making the settings, and the collimator reticule image will be again centered on the cross wires if the axes of the collimators are parallel. One of the advantages of this method of adjustment is that no recourse to external targets is necessary and it is therefore possible to set up the binocular tester without thought of getting distant external sights. The "cross-telescope" is also very useful about the laboratory whenever two collimators, however widely separated in the room, are to be brought to parallelism.

The procedure to be followed in testing a binocular for parallelism of emergent axes is obvious from the description of the instrument. When in position for this test the binocular may also be tested for magnifying power, and for differences of magnifying power in the two tubes, as well as for leaning, leaning difference, chromatic aberration, spherical aberration, position and size of reticule, and setting of dioptre movement.

Fig. 10 shows a characteristic appearance of the eyepiece field of the observing telescope with a six power glass under test. The arrangement of the collimator targets is such that the upper half of the image comes from the right hand tube of the binocular. It is readily apparent that

the quality of image produced by the left hand tube is quite faulty. The adjustment of the emergent beams is satisfactory.

When it is desired to observe a point in the field off the axis of the binocular, the instrument platform and the observing telescope are rotated in such a way as to keep the center of the collimator reticule images near the center of the field of view of the observing telescope. In this way aberrations introduced by the tester itself are no greater than when examining the axial field. By reading the circles on the instrument platform and on the observing telescope both objective and visual field angles of the binocular may be determined for any setting. From these the magnifying power for various field angles may be determined and the distortion and variation of distortion with field angle deduced if desired. Coma, if present, may be observed. By refocusing the binocular eyepiece the field curvature and astigmatic difference may be determined in terms of the dioptre scale.

OPTICAL LABORATORY,
FRANKFORD ARSENAL,
PHILADELPHIA, PA.
NOVEMBER. 1924.

**Thermionic Work-Function of oxide-coated Platinum.**—This function is determined by the two acceptable contrasting methods, upon the same filament under the same experimental conditions and at nearly the same time. By one method the quantity measured is the rate at which heat must be supplied to a filament pouring out electrons at a known rate, to compensate for the energy spent in expelling them; hence, it gives a direct value of this energy per electron. By the other method, the quantity measured is the exponent $b$ in the familiar equation relating to thermionic current and temperature, $i = AT^2\ exp\ (-b/T)$. The application of the first method, with the very elaborate and careful corrections previously set forth, yields $1.79 \pm .03$ volts at 1064°K. The second method yields 1.79 volts, provided that the curve of $i$ versus $T$ is determined point-by-point in this manner: the temperature of the filament is originally adjusted to 1064°K, and after a sufficient time, is suddenly changed to another value $T$, whereupon the new value of $i$ is *instantly* measured; and values so determined are plotted against $T$. If, however, the filament is left standing at the temperature $T$, the current from it changes with time, approaching exponentially to a limit-value; if these limit-values are plotted versus $T$, the curve displays an entirely different value of the exponent $b$, viz; 1.38 volts. The inference is, that to each distinct temperature there corresponds a distinct state of the filament and a distinct value of $b$. By whisking the temperature quickly from 1064°K to some other value and measuring the current at once, it was barely possible to keep the

old value of $b$ (the value of 1064°) long enough to get a consistent curve of $i$ versus $T$ corresponding to that one value of $b$. By waiting for the limit-values of current at each $T$ we get an entirely misleading curve. This has grave implications. Suppose, e.g., that with some other substance, the limit-value were attained so quickly in every case that the variation with time was indetectable; how should we know that the apparent value of $b$ was false, except by independent measurements with the first method? [C. J. Davisson and L. H. Germer; Phys. Rev. (2) *24*, 666-682; 1924]

KARL K. DARROW

**Bands due to the Molecules $Na_2$, $K_2$, NaK.**—The band absorption spectra of sodium and potassium have been known for a long time, and new bands in the absorption-spectrum of a mixture of the two vapors were discovered by Barratt. Analyzing two or more bands of each of these spectra, Smith identifies the $P$, $Q$ and $R$ series in each band (locating many lines of each), determines the coefficients of the parabolic equations linking the frequency to the order-number for the lines of each series, and calculates the moments of inertia of the absorbing molecules in the usual way. For potassium the moment of inertia of the molecule is $18.4 \cdot 10^{-39}$ g cm$^2$ before the quantum is absorbed and 21.2 or 21.85 (depending on the band selected for analysis) after the quantum is absorbed; for sodium very much smaller, and there seem to be two distinct initial states (?); for the supposititious NaK molecule it has an intermediate value. The calculated values for the distance between the nuclei of the atoms in the molecule are: 1.1A for Na, 3.1A for K, 2.1A for NaK. The value for K is compatible with the radius of the K ion in crystals, the value for Na is not. [S. Barratt, Leeds; Proc. Roy. Soc. *A 105*, pp. 221-225, 1924; H. G. Smith, Toronto; Proc. Roy. Soc. *A 106*, pp. 400-415; 1924]

KARL K. DARROW

**Diffusion of Mercury and Iodine Vapors through Nitrogen.**—Pretty experiments in which solid iodine is placed at one end of a long tube filled with nitrogen and liquid mercury at the other, and two contiguous ring-shaped deposits of the two solid compounds of mercury and iodine (Hg I towards the mercury end of the tube, $HgI_2$ towards the iodine end) appear immediately on the inner wall of the tube and widen laterally. From the iodine at one end of the tube to the growing ring, the concentration of iodine vapor diminishes (it is assumed) uniformly from its saturation-value to zero, and the rate at which new material is added to the ring gives the rate at which iodine flows down this gradient; simple measurements on this rate and on the distance from the ring to the end of the tube, combined with the values of vapor-pressure of iodine, give data sufficient to compute the diffusion-coefficient of iodine in nitrogen, and *mutatis mutandis* that of mercury. Extensions and interpretations are to follow. [J. M. Mullaly and H. Jacques, Oxford: Phil. Mag. *48*, pp. 1105-1122; 1924]

KARL K. DARROW

# THE DECREASE IN ELECTROMOTIVE FORCE OF UNSATURATED WESTON CELLS

By Warren C. Vosburgh

A slow decrease in the electromotive force of an unsaturated Weston cell is to be expected on account of the diffusion of mercurous-ion and its reaction with the cadmium of the amalgam electrode giving cadmium-ion and mercury. The resulting increase in the concentration of cadmium sulfate in the electrolyte must cause a decrease in the electromotive force.[1] In view of the wide use of the unsaturated Weston cell as a working standard of electromotive force it is important that the rate of decrease be known.

It has previously been found[2] that a group of 5 portable unsaturated cells decreased on the average by 0.004 per cent in 11 months, and a number of non-portable cells decreased 0.0023 per cent in 6 months. As a large number of cells set up in previous investigations[2] had been standing for a year or more since the last measurements, it was thought worth while to measure them again to arrive at a more reliable estimate of the rate of decrease.

The cells had been kept at room temperature in the dark since the previous work with them was finished. They were put in an oil thermostat at $25° \pm 0.02°$ again and measured occasionally over a period of 3 weeks. All but a few were constant within 0.01 mv from the start. The measurements were made as in the previous work, duplicate measurements of constant cells agreeing within a few microvolts. The reference standards used were Cells 1-12. These cells were the reference standards in the previous measurements of the portable cells. The standards used in the previous measurements of the non-portable cells were known to agree with Cells 1-12 as the latter were measured at the same time. Variation in the electromotive force of the standards was probably much less than 0.01 mv.[3] The results are given in Tables 1-3.

The tables show that, while all the cells decreased in electromotive force, the decrease was very slow. For the non-portable cells, Nos. 16-55, it averaged 0.04 mv, or 0.004 per cent, a year, the results for Cells 36-40, which were abnormal, being omitted. This is in good agree-

[1] Vosburgh and Eppley (J. Amer. Chem. Soc., *45*, p. 2271; 1923) have shown that the electromotive force of an unsaturated Weston cell is inversely proportional to the concentration of cadmium sulfate in the electrolyte.

[2] (a) Vosburgh and Eppley, J.O.S.A. and R.S.I., *9*, p. 69, 1924. (b) Ref. 1, p. 2273.

[3] See Vosburgh, J. Amer. Chem. Soc., *47*, p. 1256, 1925.

ment with the previous figure of 0.023 mv decrease in 6 months. There was apparently no change in the rate of decrease as the cells grew older,[4] except in the case of Cells 36-40. The portable cells decreased at the rate of 0.03 mv, or 0.003 per cent, a year. Another group of 20 portable

TABLE 1. *Non-portable Cells. Groups with Electrolytes of Different Cadmium Sulfate Concentration*

| Cells | Emf Age 8 Mo v | Emf Age 26 Mo v | $-\Delta E$ 19 Mo mv | Cells | Emf Age 8 Mo v | Emf Age 26 Mo v | $-\Delta E$ 19 Mo mv |
|---|---|---|---|---|---|---|---|
| 16-20 | 1.01824 | 1.01817 | 0.07 | 31-35 | 1.01876 | 1.01871 | 0.05 |
| 41-45 | 1.01834 | 1.01824 | .10 | 36-40 | 1.01895 | 1.01869 | ( .26) |
| 21-25 | 1.01852 | 1.01845 | .07 | 46-50 | 1.01931 | 1.01924 | .07 |
| 26-30 | 1.01868 | 1.01862 | .06 | 51-55 | 1.01965 | 1.01955 | .10 |

TABLE 2. *Portable Cells Made with Coarse Mercurous Sulfate*

| Cells | Acidity of Solution Moles per liter | Emf Age 5 Mo v | Emf Age 20 Mo v | $-\Delta E$ 16 Mo mv |
|---|---|---|---|---|
| 100-104 | 0.023 | 1.01869 | 1.01865 | 0.04 |
| 105-109 | .030 | 1.01859 | 1.01854 | .05 |
| 110-114 | .038 | 1.01860 | 1.01856 | .04 |
| 115-119 | .046 | 1.01847 | 1.01843 | .04 |
| 120-124 | .053 | 1.01864 | 1.01861 | .03 |

TABLE 3. *Portable Cells Made with Fine Mercurous Sulfate*

| Cells | Acidity of Solution, Moles per liter | Emf Age 5 Mo v | Emf Age 17 Mo v | $-\Delta E$ 12 Mo mv |
|---|---|---|---|---|
| 200-202 | 0.02 | 1.01888 | 1.01885 | 0.03 |
| 203-207 | .03 | 1.01898 | 1.01896 | .02 |
| 208-212 | .04 | 1.01906 | 1.01902 | .04 |
| 213-217 | .05 | 1.01907 | 1.01903 | .04 |

cells, the results for which are not shown, decreased 0.04 mv in a year. These cells were similar to Cells 198-217 except that 5 times as much asbestos wool was mixed with the mercurous sulfate.[5]

Among the cells of low acid concentrations there were a few abnormalities. Thus cells 36-40 showed a much greater decrease than any of the

[4] Cells 11515-11519, the rate of decrease of which was previously reported (Ref. 2a), were also measured again. Cell 11519 showed an apparent increase but was not constant. The others decreased on the average 0.04 mv in 9 months as compared with the same amount during the previous 11 months.

[5] See Ref. 2a p. 72.

other cells of Table 1 and the agreement between the individual cells of the group was very poor. Cells 198 and 199 increased in electromotive force gradually over the whole period (their values are omitted from the table) and Cell 11519 was also not constant. All of these cells had acid concentrations of 0.01 to 0.02 moles per liter. No abnormalities were noticed in cells with acid concentrations of 0.03 to 0.05 moles per liter, but two out of 4 cells with an acid concentration of 0.075 moles per liter showed decreases much larger than the average.

Of the cells with acid concentrations between 0.03 and 0.05 moles per liter the decrease for individual cells varied between 0.01 mv and 0.07 mv a year. In the large majority it was between 0.02 mv and 0.05 mv.

The factors of most influence in determining the rate of decrease must be the distance between the mercurous sulfate and the amalgam, the cross-section of the connecting tube and the presence or absence of obstructions to diffusion. In the portable cells the distance between the mercurous sulfate and the amalgam was about 9 cm and in the non-portable cells about 9.5 cm, both measured at the center of the tube. In the former there were two cork rings covered on one side with linen cloth as obstructions to diffusion while in the latter the cross arm of the H-tube was smaller in cross section than the legs.

It has previously been shown[1,2a] that the temperature coefficient and the hysteresis effects of unsaturated cells can be made very small. The above results show that with respect to constancy over long periods of time also a properly made unsaturated cell is a satisfactory working standard of electromotive force where results accurate to not more than 0.01 per cent are desired. An unsaturated cell however should be checked against a more constant standard or a recently standardized cell at least once in two years.

## SUMMARY

The electromotive forces of a number of unsaturated Weston cells were found to decrease at the average rate of 0.03 mv to 0.04 mv, or 0.003 per cent to 0.004 per cent, a year. The decrease for individual cells in which the electrolyte contained between 0.03 and 0.05 moles of sulfuric acid per liter varied between 0.01 mv and 0.07 mv a year. Some of the cells with acidities outside of this range showed greater rates of decrease, and a few required more than 3 weeks at 25° for the attainment of constancy.

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NEWPORT, R. I.

**"Why wireless electric Rays can bend round the Earth."**—Larmor postulates that free electrons, or free ions, are present in the atmosphere in a concentration which increases in the vertical direction; vibrating synchronously with the radio waves, they accelerate the propagation of these waves through the air, and as this speeding-up increases with the concentration of the ions, it increases with distance from the earth's surface, and the rays are bent around to follow the curvature of the earth. Most of the article consists of calculations to show that there is nothing intrinsically impossible about such an explanation. Thus if there were a stratum in the atmosphere one km thick, parallel to the earth's surface, in which the concentration of free electrons mounted from 0 at the bottom to 0.3 electrons per cc at the top (or say the concentration of free hydrogen ions from zero to 500), radio waves travelling in it would continuously bend their course so as just to remain within it. If, however, these electrons or ions, set into vibration by the waves, frequently bump into atoms and transfer to them the energy received from the waves instead of returning it by re-radiation to the wave train, the energy of the wave train will be rapidly removed from it and it will be gravely attenuated. It is therefore necessary to make sure that collisions of free electrons or ions with atoms are not too abundant. The amplitude of a free electron vibrating under the influence of an ordinary radio wave is much less than its mean free path; nevertheless, owing to the thermal agitation, a free electron in the lower atmosphere would encounter atoms too frequently, so that the imagined stratum of free corpuscles must be supposed to be in the upper atmosphere. The article contains a number of interesting numerical results, for example the amplitudes of a coherent light-ray of the intensity of sunlight at the earth's surface, which would be 10 volts/cm for the electric vector and .034 gauss for the magnetic vector, while the light-pressure would be 500 g/km$^2$. [J. Larmor, Cambridge; Phil. Mag. *48*, pp. 1025-1036; 1924]

KARL K. DARROW

**Rate of Decay of RaE.**—This element in disintegrating emits electrons of which the speeds are continuously distributed over a wide range, disqualifying the usual theory that most of the electrons emitted from a disintegrating element are secondary electrons expelled from the orbits by quanta proceeding from the nuclei. It therefore seems that the speed with which the electron emerges from the collapsing nucleus varies from one atom to another. If so, other differences between different nuclei of the same element might be looked for, such as a difference between the half-periods of the nuclei emitting faster electrons and the nuclei emitting slower ones. Such a difference was sought, but none was found. The half-period of RaE, a quantity of which very discordant values have been published, is fixed as 4.98 days. L. Bastings, Cavendish; Phil. Mag. *48*, pp. 1075-1080; 1924]

KARL K. DARROW

# A NEW TYPE OF TELESCOPE ESPECIALLY ADAPTED TO SPECTROMETERS

By Charles S. Hastings

If any two known glasses are selected for an achromatic combination it is found that their relative dispersive powers vary from one end of the spectrum to the other, hence a completely achromatic doublet is impossible. The prevailing practice is to make the combination as closely achromatic as possible for the particular wavelength regarded as of the greatest importance, thus there are large telescopes corrected for photographic purposes as well as others for visual work neither of which is efficient for the work of the other. When one turns to spectroscopic work where no one wavelength is of the greatest importance this inherent defect is very inconvenient. Most of the spectroscopists content themselves with an independent setting of both collimator and telescope for each region of the spectrum investigated, a few provide themselves with several pairs of objectives corrected for conveniently spaced wavelengths; in both cases the changes necessitated in passing from one portion of the spectrum to another are annoying.

For many years I have been using a new form of telescope which has a constant focal distance for all wavelengths although the focal lengths are not invariable. The advantages of such systems are obvious. When they are once adjusted they require no further changes, hence all racks and pinions, or other means of focusing, can be dispensed with as useless; only a small change of distance from the reticle to the eye lens must be provided for to allow for the chromatic aberration of the human eye, a latitude otherwise implied by facility for varying the oculars. This is the primary advantage, but the fact that all wave-surfaces leaving the collimator are plane eliminates the necessity of setting the prism of a spectroscope at minimum deviation which may be made a great convenience in designing a spectroscope for chemists' use. For example, the prism may be set for minimum deviation for the violet end of the spectrum and other portions of the spectrum brought to the center of the field by rotation of the table bearing the prism, the collimator and telescope being relatively fixed. Or again, the collimator and telescope may be fixed at a small angle and a prism of moderate angle, silvered on the back, replace the ordinary 60° prism. In either of these constructions the spectral place of an observed line can be deter-

mined from graduation on the table carrying the prism instead of by the inaccurate reflected photographed scale.

In order to guide any optician who desires to follow my experience I shall give a simple solution involving three lenses only, premising that the solution is not to be regarded as the best possible or even highly recommended, it is merely the simplest possible and thus most easily tested. Moreover, I have, for the same reason, assumed that the lenses are of negligible thickness since every optician will know how to vary the solution so as to take into account this element.

The refractive indices supplied by the optical glass makers are not of sufficient accuracy for the end in view and I therefore give the necessary constants for two types of glass which will be recognized as those most frequently employed for telescopes. Those of Type Flint are the means of accurate measures upon nine specimens of flint glass, all much alike, and the corresponding constants for Type Crown are means derived from the study of sixteen specimens of "ordinary crown." Although the determinations were not individually corrected for temperature and barometric pressure they were all taken under similar circumstance and a temperature of 70° F and a barometric height of 30 inches may be assumed.

| Line | Type Flint | Type Crown |
|---|---|---|
| C | 1.612917 | 1.516176 |
| λ5614 | 1.620264 | 1.520163 |
| F | 1.629770 | 1.525098 |
| G | 1.640796 | 1.530556 |
| h | 1.646408 | 1.533246 |

If the convention of indefinitely thin lenses is adopted, the ratios of radii of the lenses does not enter the problem, only the sums of the reciprocals of the radii for each lens. These three values may be represented by $A_1$, $A_2$ and $A_3$, the algebraic sign of a radius being positive for a convex surface. Then, for my solution—

$$\log A_1 = 8.72370 \qquad \log A_2 = 8.91570 \qquad \log A_3 = 9.21020$$

The second lens is a negative lens and is of Type Flint which is also the material of the first lens; the third lens is of Type Crown. The distance from the first lens to the second is 20.5 inches while the second and third are in contact.

With these elements a short calculation will show that the focal length for the spectral line F is 30 inches with a focal distance of 9.5

inches from the third lens. The focal distances for the spectral lines of the table are, in order—

+.0017 +.0005 +.0000 +.0006 +.0015

These quantities are clearly wholly negligible.

The fact that the focal length, and hence the magnification, increases from red to violet is of no importance in the spectroscope since the action of the telescope and collimator are compensatory, but a telescope of this type used for other purposes gives colorless images only at the center of the field. It is true that this defect may be corrected by a properly designed ocular and a large telescope of this kind might prove a superior instrument for astronomical observations, but this experiment has not yet been made.

The possible solutions with materials easily procured are very numerous. I have made and used a four lens construction with the objective a cemented doublet and in which the order of chromatic difference of magnification is reversed, but there is no evident gain over the simple type above.

If the auxiliary, or correcting, lenses are given a negative power in the collimator instead of a power zero as above, its effective focal length may be much increased with small increase in tube length. This offers a ready means of reducing the angular aperture of the collimator, when attached to a great telescope, to the proper value without materially increasing the size of the spectroscope. It also enables one to employ a much wider slit, thus yielding a spectrum free from longitudinal lines due to dust or to imperfect edges of the slit. This modification has given me much satisfaction in use.

In the collimator and telescope now used with my spectrometer the objectives are made of a high dispersive crown, of one inch aperture and thirteen inches focal length. The oculars yield magnifications of 10, 16 and 26 of which the highest is preferred when the whole aperture is effective. Of course when small prisms alone are available high powers are undesirable.

On account of the peculiar property of this system of telescopes I have been accustomed to call them orthokumatic telescopes, a name which may possibly recommend itself.

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**Photoelectric effect of potassium as function of temperature.**—Confirming the announcement of Ives (This Journal, *8*, 551) Hornbeck finds that the photosensitiveness of potassium for all wave lengths from about 430m$\mu$ to the threshold is diminished when the potassium is cooled to $-180°$. The metal, to judge from the diagram, formed a film covering part of the inner wall of a Pyrex glass tube, and had been twice distilled in vacuo; the vacuum in the tube was established by a diffusion pump; the light was obtained by dispersing the continuous spectrum of a gas-filled lamp. The temperature was measured by a pentane thermometer outside the tube. When the tube was suddenly cooled by pouring liquid air around it, the photoelectric current increased; this is attributed to condensation of potassium vapor as a fresh surface upon the film; eventually it reached a limiting value less than the value at room-temperature (10 per cent less at 440m$\mu$, 30 per cent less at 520m$\mu$, to judge from the curves). When the tube was allowed to warm up gradually the rise in photoelectric current occurred rather suddenly near $-90°$; the discontinuities in thermoelectric power and resistance, found by Bidwell near this temperature, are recalled. These measurements were reproducible if the procedure was repeated identically, but other procedures of warming or cooling produced queer results.—[J. W. Hornbeck, Cornell; Phys. Rev. *24*, pp. 631-638; 1924]

KARL K. DARROW

**The Electret.**—The electrical analogue of the permanent magnet, the *electret*, is a plate of mixed waxes and resin which, after having been melted, has slowly solidified while permeated by an intense electric field exerted between the pan in which it lies and a metal plate reposing on its surface. After this process is finished the waxen plate has an electrical polarization in the sense to be expected, which soon dies away and is supplanted by a strong permanent polarization in the opposite sense. This polarization is so strong that the electric field-strength at the surface of the plate (measured by the charge drawn by induction into an earthed plate laid upon the wax) attains 20000 volts /cm, almost the highest that could be sustained in air; and so durable that, although such processes as scraping the surfaces with a knife, melting them with a flame, washing them with acids, or irradiating them with x-rays annul the apparent surface charge temporarily, it soon returns with its original strength. The inventor seems to think that these processes neutralize the surface-charge by bringing up ions which adhere temporarily to the surface but eventually detach themselves. When the electrets are kept covered with foil they retain their apparent surface-charges "for many years" (three years seems to have been the period of observation) and even if they are left uncovered for all this time the loss they suffer is spontaneously restored when they are covered up. It is not indicated whether the inventor broke up any of the plates to see whether the polarization was diffused through the interior. [M. Eguchi, Naval College, Tokyo; Phil. Mag. *49*, pp. 179-192; 1925]

KARL K. DARROW

# METHODS FOR MEASURING THE EFFECT OF TEMPERATURE ON MAGNETIC PROPERTIES*

By Earle M. Terry

## A. HIGH TEMPERATURE

1. *Measurements between Room Temperature and the "Curie Point."* The most satisfactory method for studying magnetic properties within this temperature range is that of the Rowland ring and ballistic galvanometer, as used by Hopkinson,[1] Morris,[2] Terry,[3] and several others. Morris heated his specimen by winding about it, non-inductively, a platinum wire through which a current was passed. The primary and secondary windings, one of which served as a resistance thermometer, were also of platinum covered with thin asbestos paper, while mica strips, bent around the ring, provided insulation. When placed in an evacuated vessel containing an auxilliary glowing iron wire to absorb residual oxygen, temperatures of 1150°C were reached with an oxydation loss of only 6 per cent.

The author used a relatively small ring, e.g., 3.6 cm in diameter heated in a furnace of the Heraeus type. The ring was completely protected against oxydation by depositing upon it electrolytically a coating of copper about .7 mm in thickness. Asbestos furnished satisfactory insulation between the ring and primary winding which consisted of bare copper wire whose turns were separated by powdered magnesia moistened with alcohol. The secondary winding was of platinum, to prevent change of resistance from oxydation. Good insulation between primary and secondary is essential and was secured by winding the latter upon rings cut from magnesia plates placed above and below the specimen.

2. *Measurements above the "Curie Point."*—The method best adapted to this purpose is that of Faraday in which the magnetic susceptibility is deduced from the mechanical force exerted upon a small body placed in a non-uniform magnetic field. It has been used by

* One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. It is the purpose of these reports to present in brief outline a summary, more or less critical, of the methods which may be employed in various kinds of Physical measurements. See Editorial in this Journal, *9*, p. 410, 1924.

[1] Proc. Roy. Soc., *45*, p. 318; 1889.

[2] Phil. Mag., *44*, p. 213; 1897.

[3] Phys. Rev., *30*, No. 2, p. 133; 1909.

Curie,[4] Honda,[5] Weiss and Foëx,[6] the author[7] and others. For an object of small dimensions placed in the plane of symmetry between the poles of a magnet, the force $F$ in a direction $X$ at right angles to a field $H$ is given by the expression

$$F = m\chi H\frac{dH}{dx}$$

where $m$ is the mass of the specimen, and $\chi$ its susceptibility per unit mass. If the field strength and its gradient are measured throughout the space occupied by the specimen, the method gives results in absolute measure. For most purposes, it is better to standardize the measuring instrument by determining the force on a body whose susceptibility is known. Perhaps the most convenient procedure is to use liquid solutions of salts of some of the ferromagnetic metals such as nickel or cobalt nitrates, whose susceptibilities may be determined in absolute measure by the Quincke[8] method.

It is necessary that the specimen be placed at the same point in the field as that occupied by the standard substance. For this purpose, the point at which the force is greatest, i.e., $H\frac{dH}{dx}$ a maximum, is best suited because of its ease of location and the increased sensitivity thereby secured. It has, however, the disadvantage that, when balancing the magnetic pull against that of any arrangement in which the force changes linearly with the adjustment variable, the equilibrium is unstable. Nevertheless, by shaping the pole pieces in the manner shown in $C$ of Fig. 1, the peak of the force-displacement curve may be made fairly flat, and by restricting the motion of the system by stops, very good working conditions may be secured.

Various devices have been used to measure the force on the specimen. For example, Curie placed it on the end of a torsion arm and opposed its torque to that of a fibre carried on a divided head. Weiss and Foëx balanced the force electrodynamically by mounting the specimen on one end of a horizontal rod, the other end of which carried a coil, which, when energized by an electric current, reacted with a fixed coaxial coil. Longitudinal motion of the system was permitted by suspending it in a cradle consisting of two V shaped wire supports. For work in a vacuum,

[4] Curie, Journ. de Phys., p. 197; 1895.
[5] Honda, Ann. der Phys., *32*, p. 1027; 1910.
[6] Weiss and Foëx Archiv. de Sci. phys. et nat., *31*, pp. 4, 89; 1911.
[7] Terry, Phys. Rev. *9*, N.S. p. 394; 1917.
[8] Quincke, Wied. Ann. *24*, p. 347; 1885.

where economy of space is vital, the author has found a combination of these two methods advantageous. The arrangement is shown in Fig. 1. The specimen was supported at the proper point in the magnetic field by a slender porcelain tube mounted at the end of an aluminum

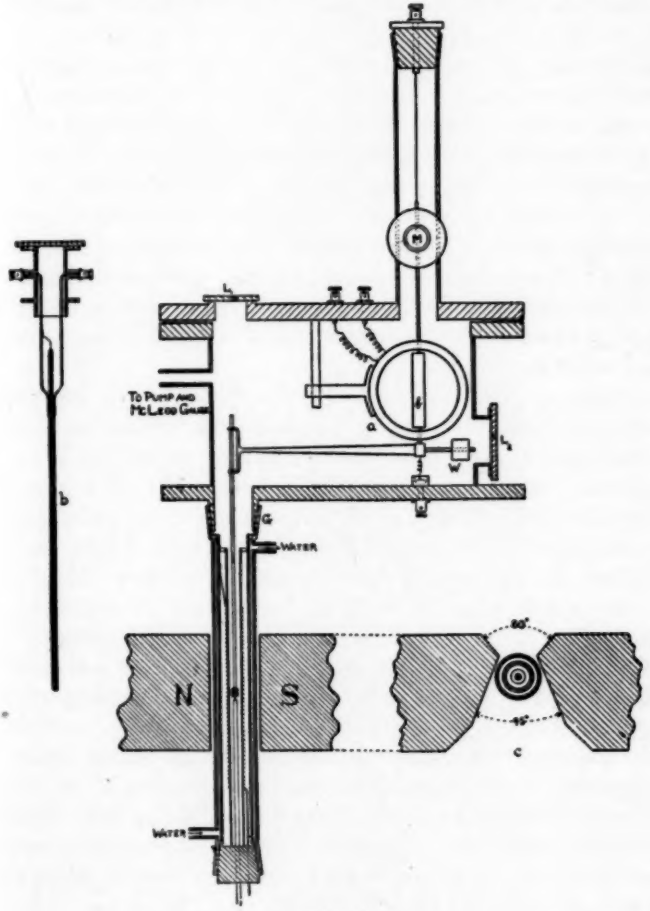



FIG. 1

torsion arm, and counterpoised by the lead nut $W$. The torque due to the field was balanced by the electrodynamic action of two coils a and b placed at right angles to each other, one of which was fixed, while the other was mounted on the movable system. A spot of light, focused by

the concave mirror *M* on a distant scale indicated when a balance had been secured.

The furnace consisted of a porcelain tube closely wound on the outside by a tungsten wire .5 mm in diameter. Outside this was another porcelain tube not shown in the figure. The furnace was mounted in a water cooled jacket supported from the bottom of the vacuum chamber by the ground joint *G*. The ordinary type of furnace employing a platinum ribbon as a heating element cannot be used in a strong magnetic field at temperatures above 1200°C since the softened ribbon is torn by the reaction of the heating current and the field. Temperature was measured by a thermocouple placed immediately below the specimen. The couple was calibrated by the fuse wire method using the arrangement shown at *b* Fig. 1, which could be inserted through the window *L*. The magnet was mounted upon a carriage resting on two rows of steel balls running in grooves and could easily be moved forward and backward to place the specimen at the point in the field where the pull was a maximum.

3. *Change of Length.*—The simplest and most direct method for measuring the magnetic change of length at high temperatures is probably that used by Honda and Shimizu[9] and is illustrated in Fig. 2. The test specimen, in the form of a rod 20 cm long and 1 cm in diameter, was threaded and brazed at each end into copper rods and supported from a tripod within an electric furnace. A weight of 2 kg. was suspended from the lower end of the system by a stout wire. The weights were immersed in water to damp out oscillations accompanying the application and removal of the magnetic field. The heating coil was wound non-inductively over a copper tube from which it was insulated by asbestos paper. A water jacket protected the magnetizing coil from the furnace. Changes of length were determined by the rotation of a cylinder mounted horizontally in jewelled bearings against which the wire supporting the weight rested with a gentle pressure. The cylinder had a series of diameters ranging from 1.5 to 2.8 mm thus providing for changes in sensitivity. Rotations of the cylinder were measured by a mirror and scale. By using the smallest cylinder diameter, elongations of $6\times10^{-8}$ cm could be measured.

4. *Wiedemann Effect.*—The twist developed by a ferromagnetic body when subjected to the combined actions of a longitudinal and circular magnetic field as a function of temperature has been studied by

[9] Honda and Shimizu, Journ. Sc. Coll. Tokyo. *19*, Art. 9; 1903-04; also Phil. Mag. *6*, sixth series, 1903, p. 392; 1903.

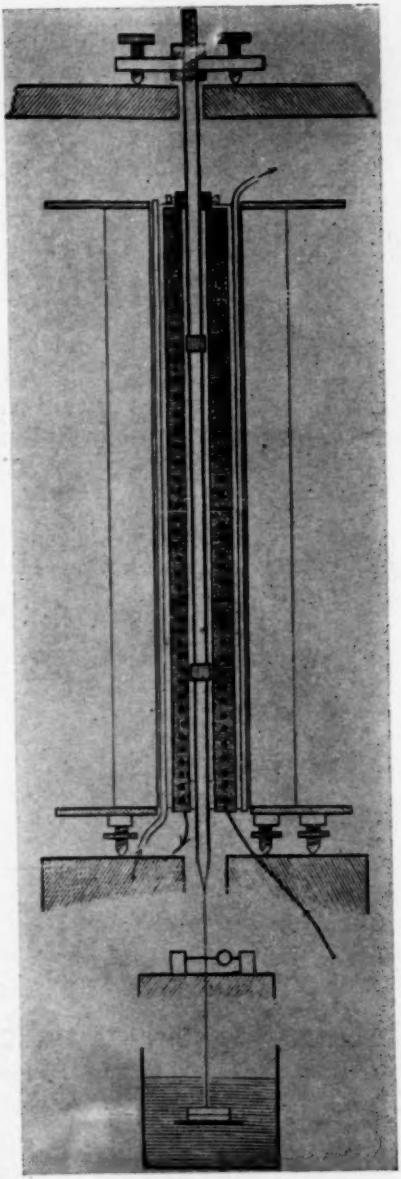


FIG. 2

Shimizu and Tanakadate[10] using the same furnace and magnetizing arrangements as described in section 3. The specimen under test was mounted in the same way except that the copper rod attached to the lower end carried a mirror and terminated in a wire dipping in a pool of mercury. The circular field was produced by sending a suitable electric current through the specimen and the twist in it due to the combined fields was measured by a telescope and scale placed at a distance of 5 meters. A rotation of .1 second of an arc was measurable with this arrangement.

### B. LOW TEMPERATURE

1. *Fields of Ordinary Magnitude.*—For fields below 150 gauss the Rowland ring method is best suited. The ring may be wound in the ordinary way and dipped or an instant in melted parafine to protect the windings against leakage due to absorption of moisture. For the lowest temperature it may be immersed directly in liquid air. For

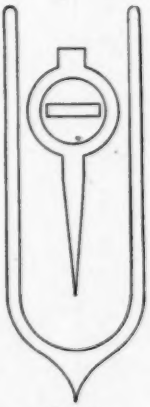


FIG. 3

intermediate temperatures the author[11] has found the following method, due to Knipp,[12] very satisfactory. A hollow copper sphere, walls about 8 mm thick, was mounted inside a Dewar flask as shown in Fig. 3. To the lower side of the sphere was attached a copper spindle which dipped into liquid air. The flask could be raised or lowered by a slow motion, while the sphere remained fixed. By properly adjusting the

[10] Proc. Tokyo Math. Phys. Soc. *3*, series 2, p. 142; 1906.
[11] Terry, Zhys. Rev. *30*, p. 141; 1910.
[12] Knipp, Phys. Rev. *15*, p. 125; 1902.

depth of the spindle in the liquid air, any desired temperature could be maintained very constant. While there was an appreciable temperature gradient from the top to bottom of the enclosure, the fall of temperature across the ring, which was placed horizontally never exceeded 2°C.

2. *Intense Fields.* For measurements in very intense fields the method devised by Weiss,[13] and used by Weiss and Kammerling Onnes[14] for determination of saturation values for iron, nickel and cobalt at liquid hydrogen temperatures is best suited. It consists in cutting the specimen in the form of an ellipsoid of revolution and measuring the torque upon it when placed with its major axis at an angle with the

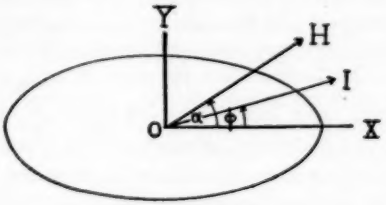



FIG. 4

field. Let the external field and intensity of magnetization make angles $\alpha$ and $\phi$ respectively with the $X$ axis, as shown in Fig. 4, and let $N_1$ and $N_2$ be the principal demagnetizing factors for the ellipsoid. Then the torque upon the specimen is given by

$$T = VIH\sin(\alpha - \phi)$$

where $V$ is the volume of the specimen. Since the susceptibility is independent of the direction, we have, using components of intensity and resultant field along the $X$ and $Y$ axes;

$$K = \frac{I\cos\phi}{H\cos\alpha - N_1 I\cos\phi} = \frac{I\sin\phi}{H\sin\alpha - N_2 I\sin\phi}$$

Eliminating $\alpha$,

$$T = VI^2(N_2 - N_1)\sin\phi\cos\phi$$

For $\phi = 45°$, the torque is a maximum, and

$$I = \sqrt{\frac{2T}{V(N_2 - N_1)}}$$

[13] Weiss, Journ. de Phys. 4th Ser. 6, p. 665; 1907.
[14] Weiss and Kammerlind Onnes, Comm. Phys. Lab. Leyden, No. 114, p. 3; 1910.

It is unnecessary to determine either $a$ or $\phi$, and the measurement may be effected by mounting the specimen between the poles of a magnet which may be rotated about a vertical axis and measuring simply the maximum torque obtainable.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
MADISON, WISCONSIN.

**Extraction of orbital Electrons from Atoms by Quanta of very high frequency.**—It has been shown by many experiments that orbital electrons may be extracted from atoms by incident quanta, emerging with kinetic energy equal to the entire energy of the quantum minus the amount required just to extract the electron. Thibaud, working in the laboratory of the Duc de Broglie, extends this work to the range of high-frequency gamma-ray quanta. Thus in the beta-ray spectrum of RaC he locates five pairs of groups of electrons, each characterized by uniform speed, the two groups of each pair differing in energy by just the energy-difference between the $K$ level and the $L$ level of the atom; adding to the kinetic energy of the electrons of each group the energy required to extract an electron from the $K$ level (or the $L$ level, as the case may be) he obtains the energy of the original quantum, and this in two cases is as high as 1763 kv, corresponding to a frequency of $4.26 \cdot 10^{20}$. These results are confirmed by measuring the energies of electrons expelled by the same gamma-rays from atoms of other metals (U, Pb, W, Ag) on which they are allowed to fall. It was indeed possible to detect electrons driven out of orbits of light atoms, and requiring only 7 to 8 kv of energy for extraction, by quanta of 500-1200 kv. This result militates against the idea that the number of electrons extracted, say from the $K$ levels of a given number of atoms by a given number of quanta of given frequency, fades away as the fourth power of the atomic number as one goes down the line from heavier atoms to lighter. [J. Thibaud, Paris; C. R. *178*, pp. 1706-1709; *179*, pp. 165-167, 815-818, 1051-1054; 1924]

KARL K. DARROW

**Linear Formulae for Vapor Pressures of various Elements.**—Linear relations between logarithm of vapor-pressure and reciprocal of absolute temperature, of the form $\log p = A + B/T$, are asserted to be quite as approximately valid for many elements as the much more elaborate thermodynamical formulae involving latent and specific heats (at least in the present status of knowledge concerning these heats). Values for the constants $A$ and $B$ are given which fit recently published data for the elements W, A, Zn, Pb, Cs, Rb and Hg. [A. C. Egerton, Oxford; Phil. Mag. *48*, pp. 1048-1054; 1924]

KARL K. DARROW

# SOME TESTS ON THE ACCURACY OF MEASUREMENT WITH THE ROTATORY DISPERSION COLORIMETRIC PHOTOMETER*

By K. S. Gibson

## INTRODUCTION

In December, 1923, a paper was published in this Journal by Irwin G. Priest on "The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion,"[1] in which were described in detail the theory, design, and method of use of the rotatory dispersion colorimetric photometer now in use at the Bureau of Standards for the color grading of light sources having colors matched by a Planckian radiator, real or hypothetical, and for the measurement of the relative intensities of such sources under conditions of exact color match, thereby eliminating the uncertainties of heterochromatic photometry.

Since its installation the instrument has been used extensively for the determination of color temperature, especially of artificial daylight units, but only recently has occasion arisen to make intensity measurements. Incidental to the work which has thus been done, certain checks on the accuracy of the measurements have been obtained. At the request of Mr. Priest, the author is publishing the present note. The data given, while in no wise exhaustive, yet constitute a rather rigid test of the performance of the colorimetric photometer.

It is assumed that the reader has the previous publication at hand and is familiar with the instrument and its use as therein set forth. The same terms and symbols will be used without redefinition.

## THE MEASUREMENT OF INTENSITY

In the paper cited above (p. 1195), the equations for relative intensity are developed only for the special experimental case in which glass plates are substituted for quartz plates in matching the comparison source with the standard. This is a simple experimental procedure and the equations are somewhat simplified. However, it is not the most general case; nor does it afford experimentally the most direct and rigorous test of the essential features of the method. If the transmission of the substituted glass plate is not exactly equal to the transmission of the quartz plate, an error is introduced.[2] The glass plates must also

* Published by permission of the Director, Bureau of Standards.

[1] J.O.S.A. & R.S.I., 7, pp. 1175-1209; 1923.

[2] Transmission is here understood as transmission for ordinary light without reference to rotatory dispersion.

be free from strain. It is therefore necessary that these plates be carefully selected and tested.

This uncertainty may be eliminated in the more general case, in which the quartz plates are left in the path while matching the standard. This is the method which has been followed in the present tests. It is experimentally practicable because there exists a value of $\phi$ for which the transmission of the rotatory dispersion system is so nearly constant with respect to wave length that it introduces no color difference.

The general equation covering all cases has previously been indicated by Priest.[3] Expressed in this general form, equations (15) and (16) of the recent publication become

$$\frac{I_x}{I_s} = k\frac{R_x}{R_s},$$

wherein $R$ represents the integral transmission of the simple or duplex filter, respectively, as given on p. 1196. Values of $R_x$ are obtained from tables as there indicated for $R$. Values of $R_s$ for the four standard conditions (p. 1196) are given in Table 1 herewith for those values of $\phi_s$ which give the closest approximation to a neutral filter. Values of $\lambda_{c,s}$ in connection with those for $R_s$ show the close approach to a spectral match which exists. Experimentally, the same comparison current is required with the quartz plates under these standard conditions as without the plates.

TABLE 1. *Standard Conditions for Intensity Measurements by Rotatory Dispersion Colorimetric Photometer*

| Source Alone | | | | $\phi_s$ or | | |
|---|---|---|---|---|---|---|
| T | $\lambda_s$ | Filter | $\phi_{1,s}$ | $\phi_{2,s}$ | $\lambda_{c,s}$ | $R_s$ |
| 2077°K | 581.02mμ | Simple | ——— | 101.8 | 581.02mμ | 1.000 |
| 2360° | 577.06 | " | ——— | 102.0 | 577.06 | 1.000 |
| 2848° | 571.92 | " | ——— | 102.0 | 571.93 | 0.999 |
| 2848° | 571.92 | Duplex | 102.0 | 102.0 | 571.94 | 0.998 |

## ACCURACY OF MEASUREMENT

I. *Color Temperature, by Means of Standard Lamps.* The following standard lamps among others are used with the instrument:

| Lamp B. S. No. | Spectral Centroid | Color Temperature |
|---|---|---|
| 3366 | 577.06mμ | 2360° |
| 3255 | 571.92mμ | 2848° |

[3] Phys. Rev. (2), *9*, p. 343; April, 1917.

Lamp No. 3366 has been matched at the Bureau with a standard lamp supplied and certified by the Nela Research Laboratory. Lamp No. 3255 has had its color temperature determined independently at the Bureau and at the Nela Laboratory with an exact check.[4]

The author has measured lamp 3255 on the colorimetric photometer in terms of 3366 at two different times (10 readings each) and obtained the following values:

| Date | Spectral Centroid | Deviation in mμ | Color Temperature |
|---|---|---|---|
| 11/22/24 | 571.95mμ | +0.03 | 2845° |
| 12/ 4/24 | 572.01mμ | +0.09 | 2839° |

These values checking within 0.1 mμ are well within the various observational errors involved.

TABLE 2. *Comparison of Observed and Computed Values of Color Temperature, Spectral Centroid, and Integral Transmission in the Case of Light at 2360° K, Transmitted by Three Blue (Daylight) Glass Filters, Nos. 1, 6 and 9. (Nov.–Dec., 1924)*

| Method | | Filter, B. S. Designation | Color Temperature | Spectral Centroid (mμ) | Deviations in mμ | Integral Light Transmission | Per Cent Deviations[d] |
|---|---|---|---|---|---|---|---|
| By Computation[a] | | 6 | 2601° | 574.30 | | $0.638_9$ | |
| | | 1 | 2701° | 573.31 | | $.560_4$ | |
| | | 9 | 2887° | 571.59 | | $.450_1$ | |
| By Flicker Photometry[b] | | 6 | | | | 0.639 | |
| | | 1 | | | | .567 | |
| | | 9 | | | | .443 | |
| By Rotatory Dispersion Colorimetric Photometer[c] | With Glass Plates | 6 | 2604° | 574.27 | −0.03 | | |
| | | 1 | 2711° | 573.21 | − .10 | | |
| | | 9 | 2927° | 571.23 | − .36 | | |
| | Without Glass Plates | 6 | 2618° | 574.12 | −0.18 | $0.618_9$ | −3.3 |
| | | 6(Repeat) | 2618° | 574.12 | − .18 | $.638_8$ | −0.1 |
| | | 1 | 2715° | 573.17 | − .14 | $.569_8$ | +1.0 |
| | | 9 | 2935° | 571.17 | − .42 | $.449_8$ | +0.7 |

[a] Computations from spectrophotometric data.

[b] Average of 6 observers, reduced to normal. Measurements by B. S. Photometry Section.

[c] 10 observations comprise a determination.

[d] Observed values minus average of computed and flicker photometry values divided by latter values and multiplied by 100.

[4] B. S. Sci. Pap. No. 443, July, 1922.

II. *Color Temperature and Intensity, by Means of "Daylight" Glass Filters.* By spectrophotometric observation and computation such filters afford a very convenient test of both color temperature and intensity measurements, the latter by means of the integral transmissions of the filters. Such comparisons of observed and computed values are summarized in Tables 2 and 3. The following points may be noted:

TABLE 3. *Comparison of Observed and Computed Values of Color Temperature, Spectral Centroid, and Integral Transmission in the Case of Light at 2848° K, Transmitted by Ives-Brady Daylight Glass (B. S. Designation, Letter of 12/2/21.)*

| Method | | Date | Color Temperature | Spectral Centroid ($m\mu$) | Deviations in $m\mu$ | Integral Light Transmission | Per Cent Deviations[c] |
|---|---|---|---|---|---|---|---|
| By Computation[a] | | 1922 | 6625° | 557.01 | | $0.154_3$ | |
| By Rotatory Dispersion Colorimetric Photometer[b] | With Glass Plates | 5/1/24 | 6740° | 556.84 | −0.17 | | |
| | | 7/18/24 | 6505° | 557.22 | + .21 | | |
| | | 11/22/24 | 6725° | 556.86 | − .15 | | |
| | Without Glass Plates | 12/ 5/24 | 6865° | 556.64 | −0.37 | $0.154_2$ | −0.0 |
| | | " | [d]( " | " ) | | $.152_1$ | −1.4 |
| | | " | [d](7070° | 556.30) | | $*.153_5$ | −0.5 |
| | | " | [d](6710° | 556.89) | | $*.151_7$ | −1.7 |
| | | " | [d](6415° | 557.40) | | $*.151_4$ | −1.9 |
| | | " | 6745° | 556.83 | − .18 | | |

[a] Computations from spectrophotometric data.

[b] 10 observations comprise a determination, except for those marked * for which but 5 observations were made.

[c] Observed minus computed values divided by latter and multiplied by 100.

[d] Color temperature and spectral centroid arbitrarily set at values given.

1. There is no consistent error in the intensity (transmission) measurements, except possibly with the duplex filter (Table 3), which small deviation may reasonably be explained by one or more of the following possibilities: (1) slight errors in the spectrophotometric determinations, (2) slight changes in the glass since the spectrophotometric measurements, (3) stray light in the colorimetric photometer.

2. The rather wide variations of color temperature shown in Table 3 are probably connected with the lack of spectral match under the

conditions of approximate color match.[5] The daylight glass has the usual green transmission band and a rather high transmission in the red beyond 650 mμ.[6] At times the color match for this particular filter seems perfect, at other times certainly not, although experimental conditions have been largely unchanged.

3. The consistent differences in color temperature and spectral centroid noted in Table 2 are within observational error, except probably for No. 9, or may likewise be explained as in the paragraphs just above. A good color match was always obtained. The fact that the color temperature measurements on the standard lamp above show deviations in the opposite sense to those with the filters indicates that no consistent error is present in the colorimetric readings.

## CONCLUSIONS

The few but rigorous tests described above serve to indicate the usefulness and reliability of the rotatory dispersion colorimetric photometer. They verify experimentally the theory on which the instrument was designed and constructed.

Inasmuch as most of the data given are for temperatures between 2360° and 3000°K, they should be of special interest to those laboratories where the measurement of the intensities of incandescent lamps of various efficiencies presents the troublesome problems of heterochromatic photometry.

WASHINGTON, D. C.
MARCH 6, 1925.

**Studies of luminous Discharges in Hydrogen between cold Anode and separately-heated Cathode.**—This article consists of descriptions of three sorts of luminous discharges in hydrogen sustained by thermionic emission from a hot-filament cathode, and measurements on the conditions at which there are sudden transitions from one form to another, with abrupt changes in current and presumably in field-distribution. Thus in hydrogen at 0.29 mm Hg the current between anode and cathode 2 cm apart leaps suddenly upward from an imperceptible value (imperceptible with the apparatus used) at 28 volts and leaps upward again at 42.5 volts, the first leap accompanying the advent of a peculiarly colored and arranged luminous discharge, the second its replacement by a discharge of a different type. The critical potentials for the advent of these discharges, and the critical potentials for their disappearance as the voltage is reduced (which are somewhat lower) depend on the electron emission from the cathode, the pressure of the

[5] Cf. J.O.S.A. & R.S.I., *7*, pp. 1181-1183; Dec. 1923.
[6] Cf. J.O.S.A. and R.S.I., *8*, pp. 201-212, Fig. 1; Jan. 1924.

hydrogen, and the distance between the electrodes. By raising the electron emission from the cathode, the authors reach a final limit-value of each critical voltage which is not further reduced by further heating of the cathode; by adjusting the pressure they find a pressure for which the critical voltage is a minimum, as for the sparking-potential in a dense gas; by adjusting the distance between the electrodes they find a gap width for which the critical voltage is a minimum, again as for sparking-potentials. This minimum of all minima is identified by special experiments, for the critical voltage of advent of the first type of discharge, with the ionizing-potential of $H_2$ near 16 volts at which intense ionization first commences. One fact hard to understand is, that as the separation of the electrodes is reduced the critical voltage of advent of the first discharge, after passing through a minimum and rising again as one would expect, passes through an unexpected maximum and declines again to something like 1.5 volts below the previous minimum. The details about the visible aspect of the discharges are best read in the original, and will be most significant to persons with first-hand experience. [O. W. Richardson and T. Tanaka (King's College); Proc. Roy. Soc. *106A*, pp. 640-662; 1924]

**Designs for the Magnification Control in Variable Power Telescopes.**—A telescope having a lens erecting system and provided with suitable adjustments has an advantage over the prismatic type with multiple ocular in that the magnification can be varied continuously between certain limits. For this it suffices to provide an erecting system composed of two converging lenses that simultaneously undergo suitable displacements. The present study shows the various combinations of movements for the two lenses that will accomplish the desired end. It is possible to have the two lenses move in the same or in opposite senses, and the combinations are arranged in two classes accordingly. The combinations with which the magnification may be varied within considerable limits are distinguished from unsuitable combinations. Curves are given for forty-seven different arrangements. From these the choice of focal data suitable for a specified case may be determined. [M. L. Dunoyer, Revue d'Optique, *3*, pp. 353-397; 1924]

G. W. MOFFITT

**The 486 mm Objective at the Strassburg Observatory.**—This objective by Merz showed astigmatism to a degree that was detrimental, although the defect had not been detected with the Hartmann test. Turning the crown through 60° improved the performance of the objective so that the principal defect was then a residual spherical aberration of the central part of the disk. A focogram of the objective shows very clearly the local defects that escaped the Hartmann test. Thus the desirability is emphasized of always including the making of a focogram with light from a star among the tests for objectives of this kind. [M. A. Danjon, Revue d'Optique, *3*, pp. 305-314; 1924]

G. W. MOFFITT

# THE NICHOLS RADIOMETER*

By J. D. Tear

The radiometer effect was first used for measuring radiant energy by its discoverer, Sir William Crookes,[1] who, although interested chiefly in the nature of the phenomenon rather than in its application, was able to measure the radiation from a candle at a distance of 19 meters. Later Pringsheim[2] used a form of the radiometer to study the near infrared region of the solar spectrum. E. F. Nichols[3] suspended two equal mica vanes upon a fine quartz fiber. The vanes, each blackened upon one side, were so adjusted as to give equal and opposite effects, thus practically compensating for stray radiation and accidental temperature variations. In this form the radiometer became an extremely sensitive and trustworthy instrument for measuring radiation. By decreasing the size and mass of the moving system Nichols obtained a further increase in sensitivity and in 1901 was able to detect and measure the radiation from a number of stars and planets.[4]

The compensating radiometer was used subsequently unchanged, save for minor modifications and developments. Saunders[5] employed a somewhat different form of case to facilitate the maintaining of constant gas pressure. Coblentz[6] successfully used narrow strips of platinum foil as vanes. Spence[7] also used metal vanes and obtained in addition to good sensitivity unusual freedom from zero drift.

The radiometer is readily applicable to the study of radiation of all frequencies, its being only necessary to provide a receiving surface which absorbs a considerable fraction of the incident energy. A thin layer of camphor smoke is a practical absorbing surface for the visible and ultraviolet and near infrared regions of the spectrum. It has been

* One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. It is the purpose of these reports to present in brief outline a summary, more or less critical, of the methods which may be employed in various kinds of physical measurements. See Editorial in this Journal *9*, p. 410; 1924.

[1] Crookes. Roy. Soc. Phil. Trans., *166*, p. 325; 1876.

[2] Pringsheim. Ann. d. Physik, *18*, p. 32; 1883.

[3] Nichols, E. F.. Phys. Rev., *4*, p. 297; 1897; Ber. Berlin Akad, p. 1183; 1896; Wied. Ann. *60*, p. 401; 1897.

[4] Nichols, E. F.: Astrophys. J., *13*, p. 101; 1901.

[5] Saunders, F. A.: Astrophys. J., *14*, p. 136; 1901.

[6] Coblentz, W. W.: Bull. Bureau of Standards, p. 404; 1907-8.

[7] Spence, B. V.: J. O. S. A. & R. S. I., p. 625; 1922.

the practice to use platinum black[8] or a mixture of india ink in water glass as an absorber for long wave infrared radiation.

Thin platinum films plated upon mica by cathode sputtering have been found effective for wave-lengths of the order of 300$\mu$ or greater.[9] There is an optimum thickness of film for each wave-length. For wave-lengths of the order of a millimeter the platinum receiving surfaces may be chosen of the proper dimensions to be in electrical resonance with the incident radiation thus securing selectivity and great sensitivity.

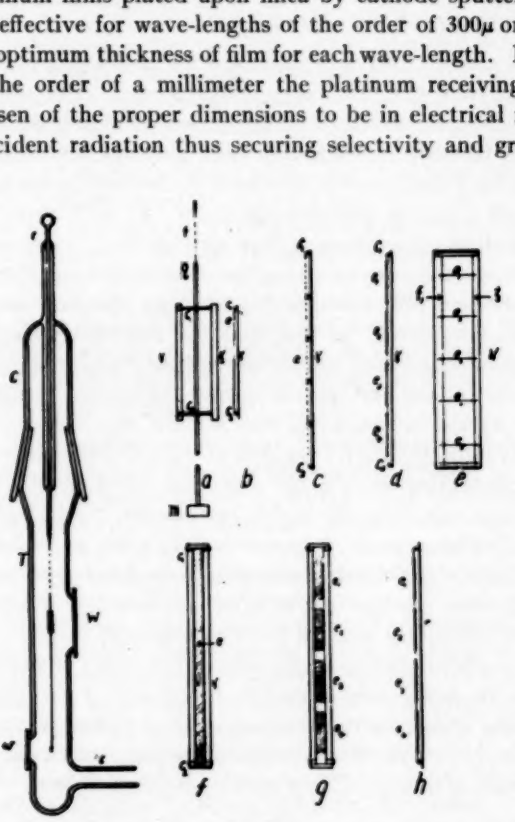



FIG. 1

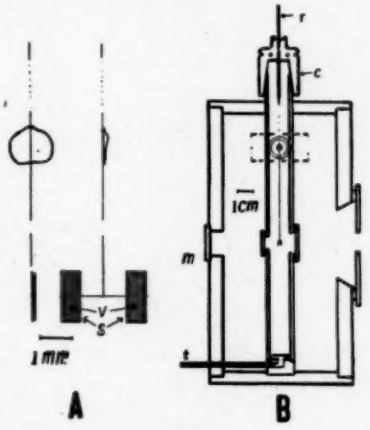
Electric wave receivers embodying the radiometer principle have been used by Hull,[10] Webb,[11] and by Nichols and Tear,[9] who after an extended investigation adopted the form shown in Fig. 1. The receiving elements *e* of b, c, d, and e are of 1$\mu$ platinum wire, f and g of narrow mica strips regionally platinized by sputtering, and h a re-

[8] Stewart: Phys. Rev., *13*, p. 262; 1901.
[9] Nichols and Tear: Phys. Rev. *21*, p. 587; 1923; Proc. Nat. Acad. Sci., *9*, p. 211; 1923.
[10] Hull: Science, *16*, p. 175; 1902.
[11] Webb: Phys. Rev. *30*, p. 192; 1910.

gionally sputtered quartz fiber. The mica shields, $V$, are so placed that the torques produced by right and left elements are additive.

With receivers of this type Nichols and Tear were able to measure radiation from a Hertzian oscillator of wave-length as short as one-fourth millimeter. They were also able to study the 324$\mu$ quartz mercury arc radiation discovered by Rubens and Von Baeyer.[12]

The success of the radiometer in the electric wave experiments together with a need for increased radiometer sensitivity led Tear[13] to a renewed study of the compensating type of instrument. The introduction of the thermally insulating shields ($S$, Fig. 2, $A$) immediately behind the receiving surfaces $V$ led to a very considerable




F.G. 2

increase in sensitivity. The shields and vanes are of mica which has been pearled by heat treatment and the front surfaces of the vanes are coated with a thin layer of camphor smoke. The shields are attached to the vanes and properly spaced by means of centrally placed minute fragments of quartz rod which are cemented with a vanishingly small quantity of fused shellac. The cross arm and axial quartz rod are about 0.02 mm in diameter and the platinum surface mirror which is chipped from a piece of optically plane glass is $1 \times 1.3$ mm with an average thickness of approximately 0.04 mm. The total weight of the suspension is about one-half milligram.

[12] Rubens and Von Baeyer: Phil. Mag., *21*, p. 689; 1911.

[13] Tear: "Improved Radiometer Construction," Phys. Rev., p. 641, No. 5, May, 1924.

Although 1×1.5 mm vanes were found to be much more sensitive than vanes 1×10 mm, further reduction in dimensions did not change the sensitivity per unit area.

One may obtain fibers which will give to such a system almost any period and at a given gas pressure the sensitivity is proportional to the period squared. However, a study of the way in which both the sensitivity and damping vary with pressure leads to the result that the best fiber is one which will give the system a half-period equal to the maximum permissible deflection time, and the best working gas pressure, that which will give critical damping. This pressure usually lies between 10 and 100 bar.

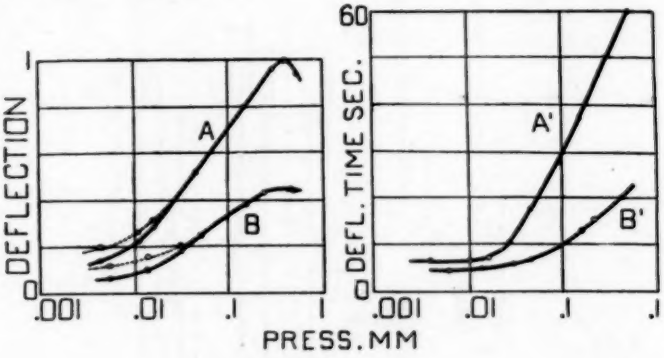



FIG. 3

The way in which the sensitivity and damping of system No. 8 Table 1 vary with pressure is shown in Fig. 3. The system was used upon two different fibers, the first giving a period of 14 sec. (curve *A*) and the second 10 sec. (curve *B*). The dotted lines represent the first turning points, and the solid lines the final steady deflections. The corresponding deflection times are shown in curves *A'* and *B'*. It may be seen that when critical damping is exceeded, the deflection time increases as rapidly as the deflection; consequently it is advisable to use the system at a pressure corresponding to critical damping and to select a fiber which will give the period chosen.

Various forms of radiometer cases have been used successfully by different investigators, the principal requirements being the possibility of maintaining a constant gas pressure and protection from local temperature variations and stray radiation. A simple form of case which has proved to be satisfactory is shown schematically in Fig. 2, *B*. The metal parts are of commercial drawn brass tubing, and the windows

are of plate glass with the exception of the one through which the radiation to be measured enters. The material chosen for this window will naturally depend on the region of the spectrum to be studied. The conical joint of the torsion head is sealed with a heavy stop-cock grease made of rubber dissolved in vaseline.

In using this type of case it has been found necessary to adjust the pressure at periods of from one to two weeks. Although the pressure and consequently the sensitivity changes slowly, if one wishes to make measurements on different days which may be compared as regards magnitude, it is necessary either to adjust the pressure accurately preceding a series of observations or to have available a comparison source by means of which the sensitivity may be determined on each occasion.

TABLE 1. *Sensitivities of Radiometers*

| Radiometer System | Vanes | Period | Candle* Sens. | Sens. per mm** 10 sec. period |
|---|---|---|---|---|
| No. 1 Nichols[4] 1901 | 2 mm dia. | 13 sec. | 724 | 136 |
| No. 2 Porter[14] 1905 | .6×6 mm | 90 | 3200 | 11 |
| No. 3 Coblentz[6] 1907 | 1×11 mm | 100 | 5700 | 5 |
| No. 4 Coblentz | .5×9 mm | 150 | 3200 | 3 |
| No. 5 Tear[13] (copy of system No. 1) | 2 mm dia. | 13 | 470 | 89 |
| No. 6 Tear (No. 5 with shields added) | 2 mm dia. | 13 | 3430 | 650 |
| No. 7 Tear | 1×10 mm | 38 | 14000 | 97 |
| No. 8 Tear | .3×1.5 mm | 14 | 2100 | 2400 |
| No. 9 Tear | .5×1.5 mm | 12 | 2600 | 2400 |
| No. 10 Tear | .5×1.5 mm | 20 | 6200 | 2100 |

* Millimeters deflection for candle and scale each at one meter.

** Estimated deflections per square millimeter exposed vane for a ten second period.

The outer case is usually wrapped with heavy felt or packed in some thermally insulating material in order to further insure constant temperature conditions.

As the accuracy of the individual observations depends to a large extent upon the freedom from mechanical as well as from thermal disturbances, the mounting of the radiometer is of importance. Basement wall brackets have been found quite satisfactory if the observations can be made at periods of the day when there is relatively little activity about the building. Such a mounting when tested with a mercury surface shows frequent tremors which however are of very

[14] Porter: Astrophys. J., *22*, p. 229; 1905.

short duration. The most satisfactory mounting has been found to be a pier built up from the bed rock and separate from the building. A mercury surface in a 15 cm evaporating dish on this pier shows only very occasional tremors and these of a duration of a small fraction of a second. A pier previously built in the same location and floated upon sand seemed to vibrate continuously.

The candle sensitivities and brief specifications of some of the more sensitive radiometers, for which data are available, are given in Table 1. The sensitivities are for a paraffin candle burning 7.7 grams per hour and for a fluorite window. In computing the values in the last column deflections were assumed proportional to the period squared, a relation found to hold accurately for the systems used by the author, though it may give too low values to the earlier long period systems which were probably much more than critically damped.

The proportionality between deflections and energy intensity has been tested for the infrared by Nichols,[3] for electric waves with a polarizing grating by Nichols and Tear,[9] and for visible radiation using rotating sector disks, by Tear,[13] and in each case found to hold well within the limits of error of the observations.

In making the last of the above tests, Tear used radiometer No. 10 (Table 1), one millimeter deflection corresponding to an energy intensity of $2\times10^{-8}$ watts per square centimeter. The deflections ranged from 100 to 2 mm and the probable error of the mean of ten observations was 0.2 mm. For the smallest deflections which could be measured with certainty under these conditions, energy was intercepted by the $0.5\times1.5$ mm vane at the rate of $3\times10^{-11}$ watts. Thus $10^{-11}$ watts seems to be the practical lower working limit of the radiometer in its present state of development.

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# SODIUM BY ELECTROLYSIS THROUGH GLASS

## RELATED EXPERIMENTS ON FARADAY'S LAWS, SODIUM ARC AND RESONANCE RADIATION

By Robert C. Burt

E. Warburg[1] tells us that Geissler tubes with electrolytically introduced sodium were shown at the international convention in Heidelberg in 1889. He describes a method of introducing sodium electrolytically through the glass in a gas discharge tube, but this method requires the use of a 1000 to 1200 volts direct current and is possible only in tubes containing gas.

With the advent of thermoionic discharge it became possible to adapt the method to the introduction of pure sodium into a very high vacuum at a rate as high as one-fourth gram metallic sodium per hour, and without the use of high voltage. The use of the incandescent lamp bulb for this purpose was brought to my attention by Dr. A. W. Hull, who, I believe, was the first to use thermoionic emission to this end and whose suggestions I wish to acknowledge.

The simplest arrangement for this electrolysis is shown in Fig. 1. The most common material used in the manufacture of the vacuum-type incandescent lamp is a soda glass in which the sodium ion is mobile.[2] If the potassium is also mobile, it is mobile to a far less degree than are the sodium ions in the glass. The method is not limited to vacuum lamps, as the author has succeeded in putting as much as 1/2 gram of sodium through the wall of a gas-filled lamp of the kind known as 50 watt mill type. However, most of the gas-filled lamps are made of a glass such as pyrex, for instance, in which the sodium ion is not mobile.

[1] E. Warburg, Wied. Ann., *21*, p. 622; 1884.
E. Warburg, Wied. Ann., *40*, p. 1; 1890.
A. Herz, Wied. Ann., *54*, p. 245; 1895.
W. P. Graham, Wied. Ann., *64*, p. 51; 1898.
C. A. Skinner, Wied. Ann., *78*, p. 754; 1899.
K. Mey, Ann. der Phys. (4), *11*, p. 130; 1903.
W. Neuswanger, Phys. Rev. (2), *7*, p. 263; 1916.

[2] For complete bibliography on conductivity of glass see "The Electrical Conductivity of Sodium Chloride in Molten Glass" by Willard J. Sutton and Alexander Silverman, Journal of the American Ceramic Society, *7*, p. 102, No. 2, Feb. 1924.

Referring to Fig. 1, *C* is a common 40 or 60 watt metal-filament lamp from which the base has been removed by heating. The filament is lighted from a 110 volt source through a resistance *R*. The bulb is partly immersed in a bath of molten sodium nitrate which is kept molten in an iron crucible *F* by a Bunsen burner *B*. An electrode *E* (a heavy copper wire is satisfactory) is also introduced into the molten salt and connected to the positive wire of the 110 volt source through a milliameter *MA* or a lamp.

If now the resistance *R* be reduced until the filament of *C* begins to emit electrons, these electrons will be carried to the glass wall opposite the molten salt by the field. These electrons neutralize sodium ions in the glass which thus become atoms. These are evaporated by the heat

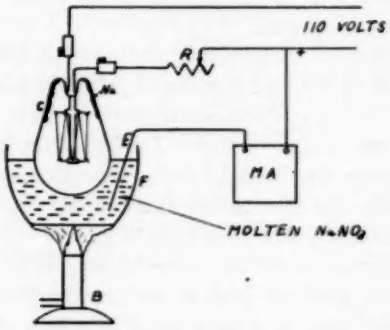


FIG. 1

at the bottom of the bulb, and condense at the top of the bulb where the glass is cooler. Sodium ions from the molten salt replace those of the glass and migrate through it, being in turn neutralized at the inner surface. In this way we are driving sodium electrolytically through the glass wall. $NO_3$ has been observed coming up from *E*.

Of course any source of sodium ions might replace the molten nitrate. Warburg used 1/2 per cent sodium amalgam. However, as the mobility of the ion in the glass increases rapidly with increase in temperature, the nitrate which melts at 312°C, is very convenient. The temperature 312°C is well below the melting point of the glass and yet sufficiently high to allow a large electrolytic current to pass through the glass with a small potential drop in the glass. Alternating current can, of course, be used in place of direct in the arrangement of Fig. 1, for the filament itself acts as a rectifier.

The field in the case of Fig. 1 is due to the potential drop in $R$ and in part of the filament and the objections to be urged against the simple connections of Fig. 1 are:—

(1) One end of the filament carries much more current than the other and becomes overheated, especially if the electrolytic current is as large as 0.1 ampere.

(2) If the entire bulb gets hot and so increases the pressure of sodium vapor, ionization is likely to start between the filament leads, burning them out and spoiling the cell.

To overcome these faults, I have used series resistances (lamp bulbs) to limit the ionization currents; also, I have applied the electrolytic field between the midpoint of the filament and the molten salt. This is accomplished by the arrangement shown in Fig. 2. The cell $C$ is connected in series with a similar light bulb $L_1$ and both filaments are lighted by 220 volts A. C. through the resistance $R$. The resistance $L_1$ now prevents too large a current through $C$, thereby removing the

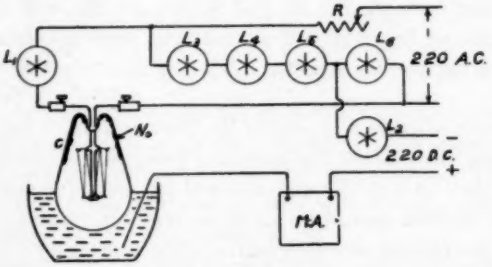


FIG. 2

second objection given above. To obtain the midpoint of the filament four light bulbs, all alike, $L_3$, $L_4$, $L_5$ and $L_6$ are connected in series across the terminals of $L_1$ and $C$. The connection between $L_5$ and $L_6$ then is at the potential of the middle of the filament in bulb $C$, and by applying the source of direct current to this point, local heating of the filament is greatly reduced. $L_2$, like $L_1$, is a lamp used as a stabilizing resistance to prevent destructive ionization current between the filament and the glass wall. $MA$ is again a current measuring device.

By the use of the connections of Fig. 2, currents as large as 0.3 amperes can be used, which results in a deposit of nearly 300 milligrams of sodium in an hour, using a 60 watt 110 volt lamp; for we have determined by experiment that this electrolysis follows Faraday's laws at least to within one part in 500. I have also found experimentally

that commercial $NaNO_3$ in the molten state does not attack the glass sufficiently to change the weight of a bulb 0.1 milligrams in several hours. For these reasons, students who are used to density-of-air weighings can make a very accurate voltameter experiment of this electrolysis. The limit of its accuracy has not yet been ascertained. The set-up as shown in Fig. 2 was used to obtain the purest of sodium surfaces for some photo-electric work which has been submitted for publication.

Apparently potassium ions will not pass through the glass, although one would expect them to replace the sodium easily. I have repeatedly tried using potassium nitrate instead of sodium nitrate around the bulb in order to get pure potassium into the bulb. The electrolytic current, at first comparable to that obtained with sodium nitrate, decreases in only a few minutes or even less, nearly to zero. If the voltage is maintained at a high value, a very small current will continue until the glass fractures. This break always occurs at a number of points in the plane of the interface between the air and molten nitrate, and the glass has the appearance of being etched all over the part in contact with the nitrate and is apparently strained. Thus it flies to pieces when put in a flame.

If the nitrate of lithium, a smaller ion than sodium, is used in place of sodium nitrate, the glass of the bulb in a few minutes closely resembles a white egg shell; it is very weak and will not hold a vacuum.

A word of caution should be given on melting sodium nitrate. Because of its mechanical strength (after being once fused), and its large thermal expansion, a geyser will be formed if it is melted at the bottom first while the surface is encrusted. To avoid this, it is desirable to melt by placing the flame against the side of the crucible. Furthermore, no water should be allowed to drop into the molten salt because steam forms quickly and spatters the nitrate which burns rather badly.

This arrangement of apparatus is also good for a lecture room experiment. Light bulbs are familiar to all. The sodium appears to go through the glass very quickly, a deposit visible to everyone in a large room appearing in a few minutes.

Ionization within the bulb has been mentioned before. It may be increased by raising the temperature of the bulb, which is easily accomplished by putting a metal hood ( *K* of Fig. 3), or an asbestos cone, over the bulb; the connections of Fig. 2 must be used for safety and stability. When the vapor pressure of the sodium is thus raised, the arc will become quite brilliant. The lines are, of course, very narrow

on account of the low pressure of the sodium vapor at the highest temperature possible in soda glass.

The narrowness of the lines in this arc make it a useful source, particularly for resonance radiation studies. No reversals have been observed. Striking evidence of the purity of the sodium is obtained from spectroscopic examination of this arc. It shows sodium lines of the principal series and also many pairs of lines of the first and also of the second subordinate series, all very distinctly, and at the same time no trace of any other line. With ionization strong enough to give such a sodium spectrum as this and no other lines at all, Dr. William R. Smythe, after observing it, tells me this is splendid evidence that potassium is not

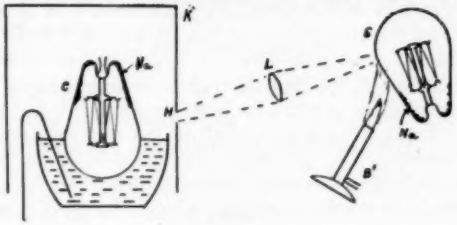


FIG. 3

present as an impurity in quantities larger than one part in 500,000. It is worthy of note that all of the foregoing results were obtained without taking any precautions whatsoever to have pure sodium nitrate. The iron crucible was not cleaned, nor was the copper electrode, and the salt used was only commercially pure.

On account of the fact that the lines which are emitted by the glowing vapor are very narrow and are apparently unreversed, it occurred to Dr. E. H. Kurth that these bulbs might prove to be a particularly efficient and convenient source of radiation for resonance experiments. The arrangement which was employed here recently to demonstrate the resonance of sodium vapor in a large lecture room is shown in Fig. 3. Light coming from a small hole $H$, in the arc container was focused on a second bulb $S$, into which sodium had previously been electrolized. When this bulb was warmed by the burner $B'$, the resonance radiation was evident at once as a strongly illuminated column along the path of the incident light.

I wish to express my thanks for the valuable criticism and help given to me by the Director of this Laboratory, Professor R. A. Millikan.

NORMAN BRIDGE LABORATORY OF PHYSICS,
PASADENA, CALIFORNIA.
DECEMBER 23, 1924.

**Striated Discharge in Mercury Vapor.**—In these experiments the data obtained with an exploring electrode are interpreted according to the principles devised by Langmuir and Schottky. The wire, thrust into the mixture of ions, electrons and neutral particles constituting the glowing gas, receives zero net current when equal numbers of electrons and positive ions strike it (or rather are absorbed in it) in unit time; but owing to the differences in mean velocity and in concentraion between the ions and the electrons, this balance occurs not when the wire is at the same potential as the surrounding space but when it is a few volts lower. The actual potential of the surrounding space is found by plotting the curve of current-into-the-wire versus potential-of-the-wire and identifying a "break" or corner in the curve, attributed to the rather sudden dissipation of the sheath of positive space-charge surrounding the wire so long as it is negative. The break is not conspicuous in all of the published curves, but is said to be confirmed by means of a hot-filament electrode, of which the potential is adjusted to just the value at which the thermionic current from it is arrested. Moving the exploring electrode step-by-step across the striations, McCurdy found the actual potential of the region, determined as described, varying cyclically and much more considerably than the potential which annuls the net current into the wire. From the sign of the field-gradient, it follows that the net space-charge is positive near the cathode-face of each striation, and negative between the striations. From the slope of the above mentioned curve and its ordinate at the "break" it is inferred (by the aid of some rather uncertain tentative assumptions) that the absolute concentrations of electrons and ions both pass through maxima near the cathodeward faces of the striations and minima between. The theoretical part of the paper is obscure and difficult to follow. [W. H. McCurdy, Princeton; Phil. Mag. *48*, pp. 898-917; 1924]

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